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A simultaneous decomposition for seven matrices with applications

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Abstract

Let $\mathbb{H}^{m\times n}$ be the set of all $m \times n$ matrices over the quaternion algebra $\mathbb{H}$. In this paper, we construct a simultaneous decomposition for seven matrices with compatible sizes: $A \in \mathbb{H}^{m\times n}, B \in \mathbb{H}^{m\times p_1}, C \in \mathbb{H}^{m\times p_2}, D \in \mathbb{H}^{m\times p_3}, E \in \mathbb{H}^{q_1\times n}, F \in \mathbb{H}^{q_2\times n}$ and $G \in \mathbb{H}^{q_3\times n}$. As applications of the simultaneous matrix decomposition, we give some solvability conditions, general solutions, as well as the range of ranks of the general solutions to the following two generalized Sylvester matrix equations $BXE + CYF + DZG = A$ and $BX + WE + CYF + DZG = A$, where A, B, C, D, E, F, and G are given quaternion matrices. Moreover, we provide some numerical examples to illustrate the results of this paper.

Keywords: Matrix decomposition, Quaternion, Sylvester matrix equation, Rank
2010 MSC: 15A03, 15A21, 15A22, 15A24

1. Introduction

In mathematics, engineering, signal, circuit and others, many problems can be transformed into the decompositions of multiple matrices (e.g. [15]-[18], [21], [59]). Paige and Saunders [41] introduced the generalized singular value decomposition of two matrices with the same row number. De Moor and Golub [19] and Zha [59] gave restricted singular value decompositions of the following general matrix triplet

$$\begin{array}{c c} & \begin{array}{cc} n & m \end{array} \\ \begin{array}{c} p \\ q \end{array} & \begin{pmatrix} A & B \\ C & \end{pmatrix} \end{array}. \tag{1}$$

Gustafson [26] presented the canonical form for the general matrix triplet (1). Chu, De Lathauwer and De Moor [9] proved that the restricted singular value decomposition of the general matrix triplet (1) can be computed using a CSD-based QR-type method. Moreover, Chu and Mehrmann [12] provided a condensed form for the following five

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