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# Numerical solution based on hybrid of block-pulse and parabolic functions for solving a system of nonlinear stochastic Itô-Volterra integral equations of fractional order

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**Abstract**

In this study, an effective numerical approach based on the hybrid of block-pulse and parabolic functions (PBPFs) is suggested to obtain an approximate solution of a system of nonlinear stochastic Itô-Volterra integral equations of fractional order. For this aim, we first introduce these functions and express some of their properties and then calculate fractional and stochastic operational matrices of integration based on these functions. Using the properties of PBPFs and obtained operational matrices, the system of nonlinear stochastic Itô-Volterra integral equations of fractional order convert to a nonlinear system of algebraic equations which can be easily solved by using Newton's method. Moreover, in order to show the rate of convergence of the suggested approach, we present several theorems on convergence analysis and error estimation which demonstrate the rate of convergence of the proposed method for solving this nonlinear system is $O(h^3)$. Finally, two examples are included to illustrate the validity, applicability and efficiency of the proposed technique.

**Keywords**: Parabolic functions; Block-pulse functions; Stochastic integral equations; Brownian motion process; Fractional calculus; Operational matrix.

**AMS Subject Classification**: 60H20, 65R20, 45D05, 26A33, 40C05.

## 1 Introduction

Various problems in different branches of science are modeled by stochastic integral equations, stochastic differential equations, and stochastic integro-differential equations. In fact, many problems in mechanics, medical, physics, applied mathematics, engineering and social sciences [1-4] and option valuation problems in quantitative finance [5, 6, 7] can be modeled by using stochastic functional equations.

In most cases, these equations can not be solved explicitly or finding their analytic solution is very difficult. Therefore, researchers have applied different methods to obtain numerical solutions of stochastic differential equations or stochastic integral equations that we refer here to some of them. Runge-Kutta method [8], meshless method [9, 10], truncated Euler-Maruyama method [11], Finite difference scheme [12], wavelet-based computational method [13] and operational matrix method [14, 15, 16, 17, 18] are some of the techniques which have been utilized to solve stochastic functional equations.

In this paper, we consider a system of nonlinear stochastic Itô-Volterra integral equations of fractional order. Many problems arising in various sciences such as engineering, biomathematics, and physics lead to the linear or nonlinear system of Volterra integral equations. For example, many problems for neural networks, thermo-elasticity problems and some heat transfer problems in physics are modeled by a system of Volterra integral equations [19, 20, 21]. There are several methods for solving the linear or nonlinear system of Volterra integral equations such as Adomian-Pade technique [22], block by block method [23], Variational iteration method [24] and Homotopy perturbation method [25].

The theory of fractional calculus plays an important role in the mathematical modeling of phenomena by fractional differential equations, fractional integral equations and fractional partial differential equations. In other words, the behavior of many systems can be described by linear and nonlinear systems of fractional differential equations and fractional integral equations. Several methods are proposed to solve many classes of fractional equations that we only refer to Adomian decomposition method [26], homotopy perturbation method [27], operational matrix of generalized hat functions [28], collocation method [29], operational matrix of piecewise linear functions [30] and Galerkin method [31].

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