Accepted Manuscript

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PII: S0377-0427(18)30565-X
DOI: https://doi.org/10.1016/j.cam.2018.09.020
Reference: CAM 11915

To appear in: *Journal of Computational and Applied Mathematics*

Received date : 5 December 2017
Revised date : 5 September 2018

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Journal of Computational and Applied Mathematics 00 (2018) 1–17

Identification of a source in a fractional wave equation from a boundary measurement.

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Abstract

We deal with an inverse source problem in a time-fractional wave equation. The time-dependent source term is reconstructed using the additional non-invasive measurement in the form of integral over a part of the boundary. We look for the solution and the source term obeying the variational formulation and the equation obtained from applying the measurement on the equation. Using the Rothe method the existence of the solution is proved. The uniqueness is shown in appropriate spaces and some numerical experiments are presented.

Keywords: time-fractional wave equation, inverse source problem, boundary measurement, reconstruction, convergence, time discretization

1. Introduction

In this article, we are interested in the following fractional wave equation accompanied with standard initial condition and the Neumann boundary condition

$$\begin{cases} \left(g_{2-\beta} * \partial_{tt} u(x)\right)(t) - \Delta u(x,t) = h(t)f(x) + F(x,t), & x \in \Omega,\ t \in (0,T), \\ u(x,0) = u_0(x), & x \in \Omega, \\ \partial_t u(x,0) = v_0(x), & x \in \Omega, \\ -\nabla u(x,t) \cdot \boldsymbol{\nu} = \gamma(x,t), & x \in \Gamma, t \in (0,T), \end{cases} \tag{1.1}$$

where $\Omega \subset \mathbb{R}^d$ is bounded with the Lipschitz boundary Γ (cf. [1]), the symbol $\boldsymbol{\nu}$ denotes the outer normal vector assigned to the boundary, $T > 0$, $g_{2-\beta}$ is the Riemann-Liouville kernel given by

$$g_{2-\beta}(t) = \frac{t^{1-\beta}}{\Gamma(2-\beta)}, \qquad t > 0,\ 1 < \beta < 2,$$

and $*$ is a convolution on the positive half line

$$(k * v)(t) = \int_0^t k(t-s)v(s)\,\mathrm{d}s.$$

The convolution term

$$\partial_t^\beta u(x,t) = \left(g_{2-\beta} * \partial_{tt} u(x)\right)(t)$$

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