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K. Šišková, M. Slodička

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Identification of a source in a fractional wave equation from a boundary measurement.

K. Šišková, M. Slodička

Department of Mathematical Analysis, research group of Numerical Analysis and Mathematical Modeling (NaM²),
Ghent University, Gent 9000, Belgium

Abstract

We deal with an inverse source problem in a time-fractional wave equation. The time-dependent source term is reconstructed using the additional non-invasive measurement in the form of integral over a part of the boundary. We look for the solution and the source term obeying the variational formulation and the equation obtained from applying the measurement on the equation. Using the Rothe method the existence of the solution is proved. The uniqueness is shown in appropriate spaces and some numerical experiments are presented.

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Keywords: time-fractional wave equation, inverse source problem, boundary measurement, reconstruction, convergence, time discretization

1. Introduction

In this article, we are interested in the following fractional wave equation accompanied with standard initial condition and the Neumann boundary condition

$$\begin{cases} (g_{2-\beta} * \partial_t u(x))(t) - \Delta u(x, t) = h(t)f(x) + F(x, t), & x \in \Omega, t \in (0, T), \\ u(x, 0) = u_0(x), & x \in \Omega, \\ \partial_t u(x, 0) = v_0(x), & x \in \Omega, \\ -\nabla u(x, t) \cdot \nu = \gamma(x, t), & x \in \Gamma, t \in (0, T), \end{cases} \quad (1.1)$$

where $\Omega \subset \mathbb{R}^d$ is bounded with Lipschitz boundary Γ (cf. [1]), the symbol ν denotes the outer normal vector assigned to the boundary, $T > 0$, $g_{2-\beta}$ is the Riemann-Liouville kernel given by

$$g_{2-\beta}(t) = \frac{t^{1-\beta}}{\Gamma(2-\beta)}, \quad t > 0, \quad 1 < \beta < 2,$$

and $*$ is a convolution on the positive half line

$$(k * v)(t) = \int_0^t k(t-s)v(s) \, ds.$$

The convolution term

$$\partial_t^\beta u(x, t) = (g_{2-\beta} * \partial_t u(x))(t)$$

Email addresses: katarina.siskova@ugent.be (K. Šišková), marian.slodicka@ugent.be (M. Slodička)
URL: <http://cage.ugent.be/~ms> (M. Slodička)

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