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Crank-Nicolson and Legendre spectral collocation methods for a partial integro-differential equation with a singular kernel

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Abstract

In this paper we present an efficient numerical method for the solution of a partial integro-differential equation with a singular kernel. In the time direction, a Crank-Nicolson finite difference scheme is used to approximate the differential term and the product trapezoidal method is employed to treat the integral term. Also for space discretization we apply Legendre spectral collocation method. We discuss the stability and convergence of proposed method and show that the method is unconditionally stable and convergent with order $\mathcal{O}(\tau^{\frac{3}{2}} + N^{-s})$ where τ, N and s are time step size, number of collocation points and regularity of exact solution respectively. We compare the numerical results of proposed method with the results of other schemes in the literature in terms of accuracy, computational order and CPU time to show the efficiency and applicability of it.

Keywords: Partial integro-differential equation, Legendre spectral collocation, Stability, Convergence, Singular kernel.

1 Introduction

Integral equation has been one of the essential tools for various areas of physics and applied mathematics. Many mathematical formulations of physical phenomena contain integro-differential equations.
In this paper we study the numerical solution of following partial integro-differential equation with a weakly singular kernel

$$u_t(x,t) = \gamma u_{xx}(x,t) + \int_0^t (t-s)^{-1/2} u_{xx}(x,s)\, ds, \qquad (x,t) \in \Omega, \tag{1.1}$$

where $\Omega = \{(x,t) | -1 \le x \le 1,\ 0 < t \le T\}$, $\gamma \ge 0$, boundary conditions are

$$u(-1,t) = u(1,t) = 0,$$

and initial condition is

$$u(x,0) = g(x).$$

This type of equations have widely occurred in the mathematical modeling of various physical phenomena, such as heat conduction in materials with memory, combined conduction, convection and radiation problems [11, 16], phenomena associated with linear viscoelastic mechanics

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