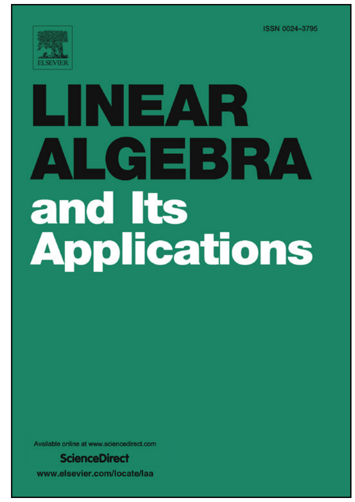


# Unification of the arithmetic-geometric mean and Hölder inequalities for unitarily invariant norms

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# Unification of the arithmetic-geometric mean and Hölder inequalities for unitarily invariant norms

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**Abstract**

In this paper, we obtain an inequality for unitarily invariant norms which unifies the arithmetic-geometric mean inequality and the Hölder inequality for unitarily invariant norms.

*Keywords:* Unitarily invariant norms; Arithmetic-geometric mean inequality; Hölder inequality
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## 1. Introduction

Let $M_n$ be the space of $n \times n$ complex matrices. Let $s_1(A) \geq \cdots \geq s_n(A) \geq 0$ be the singular values of $A \in M_n$. Let $\|\cdot\|$ denote any unitarily invariant norm on $M_n$. If all the eigenvalues of $A \in M_n$ are real, then we label them as $\lambda_1(A) \geq \cdots \geq \lambda_n(A)$.

Let $A, B \in M_n$. Thirty years ago, Bhatia and Kittaneh [11] proved an arithmetic-geometric mean inequality for singular values

$$s_j(A^*B) \leq s_j(AA^* + BB^*), \; j = 1, \cdots, n,$$

which implies

$$\|A^*B\| \leq \frac{1}{2}\|AA^* + BB^*\|. \tag{1.1}$$

This is the arithmetic-geometric mean inequality for unitarily invariant norms.

Let $A, X, B \in M_n$. Bhatia and Davis proved in [9] that

$$\|A^*XB\| \leq \frac{1}{2}\|AA^*X + XBB^*\|. \tag{1.2}$$

which is a generalization of (1.1). As pointed out in [10], the insertion of $X$ is no idle generalization, a judicious choice can lead to powerful perturbation theorems.

After that, many authors discussed different proofs, equivalent statements, generalizations, refinements, and applications of inequalities (1.1) and (1.2). For more information on this topic, the reader is referred to [12, 15] and the references therein.

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