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Kepler: Analogies in the search for the law of refraction

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ABSTRACT

This paper examines the methodology used by Kepler to discover a quantitative law of refraction. The aim is to argue that this methodology follows a heuristic method based on the following two Pythagorean principles: (1) sameness is made known by sameness, and (2) harmony arises from establishing a limit to what is unlimited. We will analyse some of the author's proposed analogies to find the aforementioned law and argue that the investigation's heuristic pursues such principles.

Kepler never hid his sympathy for Pythagoras. His knowledge of Pythagoras came from Aristotle and from Proclus' judicious reception of Euclid's work. On another occasion, I suggested that the Keplerian methodology is inspired by two principles of Pythagorean origin: (i) sameness is made known by sameness, and (ii) harmony arises from establishing a limit to the unlimited. The methodological influence of Pythagoras can be summarized by what I call the Keplerian *Leitmotiv*. This *Leitmotiv* can be synthesised into the following stages:

(1) *Problem formulation.* Many of the problems of natural philosophy addressed by Kepler conform to the following natural form: given that phenomena in a certain restricted field exhibit a regularity such that despite the fact that we could expect infinite logical possibilities, only a small number of these possibilities are present, we could thus reach the conclusion that there is a profound metaphysical reason that explains why the possibilities have been restricted in such a manner.[1] This metaphysical necessity is intimately linked with the assumption of order and harmony in the world, which implies, in the Pythagorean sense, that a certain limit is imposed on the unlimited.

(2) *Search for a contrast analogy.* Given the problem, the researcher must recognise that the foundation of the assumed harmony is in some way hidden from him. We can imagine, in a Pythagorean sense, that mathematics (especially geometry) provides a tool that renders the harmony underlying the problem obvious. Thus, the researcher can proceed to seek a mathematical analogy that can be juxtaposed with the problem situation by searching for (a) a mathematical resource that provides a finite number of control rules for the framework of infinite possibilities and (b) a resource that engages in a familiar manner with the problem situation. An analogy is a finitistic instrument of control that allows us to grasp the relations that determine the imposition of a limit on what is unlimited.

(3) *Deployment of obstructions.* The creative power of the researcher resides in providing an adequate analogy. Analogies are never coupled with absolute ease. In fact, analogies allude to absolutely simplified ideal situations. Thus, it is not strange that the deployment of finitistic control criteria applied on a mathematical instrument produces results that differ from the natural circumstances in which the world's information is collected. Once the researcher faces obstructions, as long as he does not abandon the potential he sees in the analogy, he should proceed or make adjustments to the

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[1] By a metaphysical reason I allude to a cause that explains why things could not be otherwise (demonstration of the reasoned fact: *propter quid*). Research must derive effects from causes; not derive causes from effects. The metaphysical cause imposes recognition of God's transcendent presence. Charlotte Methuen sums up the importance of maintaining the presence of God on the horizon of research: «Kepler's own work, [...], confirmed him both in his conviction that nature [...] could reveal God in a special way and in his assurance that these "truths" of nature can and will be revealed» (1998, p. 209).

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analogy to achieve a more fine-grained coupling or to find material circumstances that explain why this adjustment cannot be made.

(4) *Gathering results.* Constant research in the aforementioned direction can provide three types of results. (a) An ultimately successful coupling: in this case, the research project reaches its goal with the expected results. (b) A coupling, although truncated, provides unexpected results: the analogy does not achieve successful finite control rules that can reproduce the regularities, but in the exercise of examining the couplings, we achieve new regularities that we possibly would not have accessed if not for our stubborn investigation. (c) A coupling that hints at unattainability: the researcher decides to abandon what seemed to be a promising analogy.

In Chapter 4 of *Paralipomena*[2] Kepler brilliantly summarizes what we call the *Leitmotiv* of the Keplerian methodology. Consider the passage:

> *For geometrical terms ought to be at our service for analogy. I love analogies most of all: they are my most faithful teachers, aware of all the hidden secrets of nature. In geometry in particular they are to be taken up, since they restrict the infinity of cases between their respective extremes and the mean with however many absurd phrases, and place the whole essence of any subject vividly before the eyes*. (*Paralipomena*, p. 109; *GW*, II, p. 92).

When the investigator of nature is confronting a problem, he assumes nature hides a key that does not emerge naturally on the surface. Kepler recommends comparing the problematic situation with an analogy. The researcher restricts the endless logical possibilities to a reduced set of possibilities. Finding a geometric analogy means finding a mathematical resource that provides a system of finite control over the infinite and that, aside from the differences, offers the same behaviour on the surface as that exhibited by the particular aspect of nature.

As D. Walker points out: «*Harmony, musical or of any other kind, consists in the mind's recognizing and classing certain proportions between two or more continuous quantities by means of comparing them with archetypical figures*» (1978, pp. 44–45). For this reason, Walker believes that Kepler would prefer geometric to arithmetic analogies. In fact, Walker writes: «*Analogies based purely on numbers correspond to no archetype in the soul of man or mind of God, whereas geometric analogies do so correspond, and, in many cases, are therefore more than analogies: they display the reasons why God created things as they are and not otherwise*» (1978, p. 44).

Proclus, who strongly influenced Kepler, provides a recommendation quite akin to Kepler's methodological order. We quote him in full:

> *Mathematicals are the offspring of the Limit and the Unlimited, but not of the primary principles alone, nor of the hidden intelligible causes, but also of secondary principles that proceed from them and, in cooperation with one another, suffice to generate the intermediate orders of things and the variety that they display. This is why in these orders of being there are ratios proceeding to infinity, but controlled by the principle of the Limit*. (trans. 1970, p. 5).

Later Proclus adds « *And certainly beauty and order are common to all branches of mathematics, as are the method of proceeding from things better known to things we seek to know and the reverse path from the latter to the former, the methods called analysis and synthesis*» (trans. 1970, p. 6–7). Proclus, in effect, anticipated the Keplerian *Leitmotiv.*[3]

The main explicit references made by Kepler to Proclus are posterior to the *Paralipomena*. This fact, as pointed out by one of the reviewers of this article, casts doubt on the early influence of Proclus on Kepler. However, the idea of imposing a limit on the unlimited by means of a mathematical instrument was already present in the *Mysterium Cosmographicum* when Kepler suggested that regular solids embedded in spheres respond to the question "Why are there six planets when there they could be many more?" In addition, the first printed Greek text of the commentary on Euclid by Proclus was edited by Symon Grynaeus, who was in Tübingen in 1534 and 1535 to participate in curriculum reform at the university where Kepler has studied. Both Grynaeus and Philip Melanchthon helped to disseminate Proclus' ideas in German universities. Melanchthon had great influence among Kepler's professors, among them Jacob Heerbrand. In 1602 Kepler wrote to David Fabricius: "I have written against Ursus, but it does not satisfy me; I must first read Proclus and Averroes on the history of hypotheses" (quoted in N. Jardine, 1988, p. 28). So although we can't be sure that Kepler read Proclus before taking up *Paralipomena*, the preceding arguments suggest that this possibility cannot be completely ruled out.[4]

Gerd Buchdahl suggests that the use of analogies (archetypes, the author says) function in the manner of regulative principles.[5] In principle, I do not feel comfortable with this recommendation. I can think of only two ways of understanding law as a regulative principle. First, it cannot be a law that describes a family of phenomena, but a principle for constructing such laws. This occurs with the principle of conservation of energy or the principle of minimal action, for example. Secondly, there may be a law prescribing the meaning of a concept that we want to introduce, but rather than doing so in an explicit way it presents the meaning at the same time that the law is established. This occurs, for example, with respect to Newton's first law. Having said that, in my perspective, the use that Kepler gives to analogies is not related to either of the meanings that I see for a regulative principle. As we will see, analogies do not establish the form that we would like a law to take nor do they introduce new concepts to the system. I will demonstrate that the mentioned analogies function as control instruments that we are able take up in complex cases (cases involving the presence of

[2] After the death of Tycho Brahe (1601), Kepler dedicated part of his time to conceiving and writing *Ad Vitellionem paralipomena, quibus Astronomiæ pars optica traditur* (1604). This work, which hereinafter I will abbreviate as *Paralipomena*, was written in the form of critical commentary on the optics of Witelo and ultimately became the origin of a fundamental revolution in the study of optics.

[3] Kepler transcribed as epigraph a Proclus' passage in Book III of *Harmony*. A part of this epigraph says: «*Thus Plato teaches us many remarkable things about the nature of the gods through the appearance of mathematical things; and the Pythagorean philosophy disguises its teaching on divine matters with these, so to speak, veils*» (1619/1997, p. 127) (cfr. Proclus, trans. 1970, p. 19). In Zaiser's words: «*Harmony is present when a multitude of phenomena is regulated by the unity of a mathematical law which expresses a cosmic idea*» (1932, p. 47). I am grateful for the comments of one of the readers of this text, who warned of the danger of bringing Kepler in an amiable relationship with numerology. I want to clarify that the Pythagoreanism, attached to only the two above-mentioned principles, is, rather, the Pythagoreanism of Proclus. Walker said rightly that Kepler agreed with the criticism of Aristotle against the Pythagorean number; however he stresses that Kepler was in accordance with the Proclus' philosophy of continuous quantities (1978, p. 44).

[4] Ch. Methuen (1998) presents a full study of the intellectual environment in Tübingen at the time when Kepler studied there.

[5] Buchdahl, however, also endows analogies with a function tied to justification. The second use may be closer to what I want to defend here. According to the author says: «Methodologically, they [the archetypes, or analogies] act as necessary rules, regulative maxims; whilst epistemologically, they function as principles of justification» (G. Buchdahl, 1972, p. 276).