Studies in History and Philosophy of Science 50 (2015) 28–37

Contents lists available at ScienceDirect

Studies in History and Philosophy of Science

journal homepage: www.elsevier.com/locate/shpsa

Maxwell, Helmholtz, and the unreasonable effectiveness of the method of physical analogy

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ARTICLE INFO

Article history:
Available online 11 November 2014

Keywords:
Models;
Analogies;
Maxwell;
Helmholtz;
Structuralism

ABSTRACT

The fact that the same equations or mathematical models reappear in the descriptions of what are otherwise disparate physical systems can be seen as yet another manifestation of Wigner's "unreasonable effectiveness of mathematics." James Clerk Maxwell famously exploited such formal similarities in what he called the "method of physical analogy." Both Maxwell and Hermann von Helmholtz appealed to the physical analogies between electromagnetism and hydrodynamics in their development of these theories. I argue that a closer historical examination of the different ways in which Maxwell and Helmholtz each deployed this analogy gives further insight into debates about the representational and explanatory power of mathematical models.

When citing this paper, please use the full journal title *Studies in History and Philosophy of Science*

1. Introduction: Wigner's Puzzles

Eugene Wigner, in his classic paper "The Unreasonable Effectiveness of Mathematics in the Natural Sciences," poses two challenges: The first concerns the subject of his title most directly, namely the challenge of understanding how "mathematical concepts turn up in entirely unexpected connections" (Wigner, 1960, p. 2). The second challenge is how we can "know whether a theory formulated in terms of mathematical concepts is uniquely appropriate" (p. 2), or what he later describes as the remarkable accuracy and (prima facie) explanatory power of false theories. There is, however, a third puzzle that lies at the intersection of Wigner's two challenges, and that is understanding how the same equations or mathematical models can sometimes reappear in the descriptions of what are otherwise very different sorts of physical systems. This puzzle not only raises questions about the unreasonable effectiveness of mathematics but also Wigner's worries about non-uniqueness and the prima facie explanatory power of false models. Moreover, this formal similarity between two distinct domains of science, can give rise to a methodology whereby the results obtained in elaborating the models in one domain can then be imported into the other domain to also solve problems there.

This third puzzle, involving the same mathematical equations reappearing in the descriptions of what are otherwise very different physical systems, is most strikingly illustrated in the works of James Clerk Maxwell and Hermann von Helmholtz. Maxwell famously exploited these formal similarities between two distinct domains of science in what he called the method of physical analogy. An early articulation of this methodology occurs in his 1855 article "On Faraday's Lines of Force", where he writes,

> By a physical analogy I mean that partial similarity between the laws of one science and those of another which makes each of them illustrate the other. . . . [W]e find the same resemblance in mathematical form between two different phenomena. (Maxwell [1855/56] 1890, p. 156)

Maxwell used this methodology repeatedly in his development of the theory of electromagnetism, such as by drawing physical analogies between fluid dynamics (hydrodynamics) and electromagnetic phenomena. He, for example, conceives of Faraday's lines of force as thin tubes carrying an imaginary incompressible fluid, though explicitly notes that this fluid should not be thought of as a physical hypothesis, but rather simply as a useful fiction.

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http://dx.doi.org/10.1016/j.shpsa.2014.09.012

Interestingly Helmholtz independently exploited this very same physical analogy; but rather than using hydrodynamics for the further development of electromagnetism, Helmholtz used electromagnetism for the further development of hydrodynamics.[1] In his seminal 1858 paper on vortex motions, he writes that there is a

> remarkable analogy between the vortex-motion of fluids and the electro-magnetic action of electric currents. . . . I shall therefore frequently avail myself of the analogy of the presence of magnetic masses or of electric currents, simply to give a briefer and more vivid representation. (Helmholtz, 1858, p. 43; 1867, pp. 486–487)

By using this physical analogy and fictitious representation, Helmholtz was able to derive three fundamental theorems of fluid dynamics that are still accepted today.[2] While the fertility of this method of physical analogy is, I believe, indisputable, its philosophical grounding and implications are still not fully understood. To many, Maxwell's and Helmholtz's remarkable successes using this method are just another indication of Wigner's "unreasonable effectiveness of mathematics."

In what follows I shall take a closer look at the different ways in which Maxwell and Helmholtz each deployed this physical analogy between hydrodynamics and electromagnetism, and offer a more nuanced historical understanding of how this methodology works. My aim in this paper is not to reconstruct Maxwell's logic of scientific discovery (for this see, for example, Buchwald's (1985) rigorous and detailed book); rather, my aim is to use Maxwell's own reflections on the method of physical analogy as a framework for thinking about Wigner's puzzles and the representational power of mathematics. Maxwell does not simply employ these physical analogies and fictional posits with a naive opportunism, but rather engages in a philosophical reflection on both the legitimacy of such a methodology and its broader metaphysical implications. There are three points in Maxwell's reflections on this methodology that I wish to call attention to: The first concerns Maxwell's views on how mathematical models represent reality; the second, his views on the explanatory power of mathematical models; and the third, the version of scientific structuralism that Maxwell believes underlies this method of physical analogy. I shall conclude by showing the relevance of Maxwell's reflections for Wigner's puzzles and what I call "the unreasonable effectiveness of the method of physical analogy."

2. Maxwell's method of physical analogy

As Maxwell himself describes it, the most immediate source of inspiration for his method of physical analogy, was William Thomson's (Lord Kelvin) use of the analogy between heat and electrostatics, as presented in an 1842 paper.[3] In a letter to Thomson in the spring of 1855 Maxwell acknowledges this influence:

> I am trying to construct two theories, mathematically identical, in one of which the elementary conceptions shall be about fluid particles attracting at a distance while in the other nothing (mathematical) is considered but various states of polarization tension &c existing at various parts of space. The result will resemble your analogy of the steady motion of heat. Have you patented that notion with all its applications? for I intend to borrow it for a season without mentioning anything about heat … but applying it in a somewhat different way to a more general case. (Maxwell letter to Thomson, 15 May, 1855; in Harman, 1990, pp. 306–307).[4]

As we will see, Maxwell greatly expanded Thomson's method of physical analogy, using it in a more general way. Nancy Nersessian explains the difference between Thomson's and Maxwell's use of the method of analogy as follows:

> Thomson's method was to take an existing mathematical representation of a known physical system … as an analogical source….That is, Thomson proceeded directly to the mathematical structures using a formal analogy between the two real-world domains…. What makes Maxwell's [approach]… different is that the analogical sources to be mapped to the domain of electromagnetism were not ready to hand, but had to be constructed. (Nersessian, 2008, p. 51)

This more creative use of the method of analogy has been remarked on by many, such as Giora Hon and Bernard Goldstein who argue that it anticipated a very modern approach to modeling:

> Unlike Thomson, Maxwell described an artifact–an imaginary scheme–which he set into an analogical relation with the newly discovered electromagnetic phenomena…. this shift constitutes a new methodology: the application of contrived analogy, which may be considered the harbinger of the modern methodology of modeling. (Hon & Goldstein, 2012, p. 246)

In order to better understand these innovations, let us turn to a closer examination of Maxwell's method.

Maxwell introduces his physical analogy as a middle path between what he calls a "purely mathematical formula" on the one hand and a "physical hypothesis" on the other.[5] He notes that if one adopts a purely mathematical approach, conceiving of these equations as nothing more than a string of mathematical symbols, then "we entirely lose sight of the phenomena to be explained; and though we may trace out the consequences of given laws, we can never obtain more extended views of the connexions of the subject" (Maxwell [1855/56] 1890, p. 155). According to Maxwell, an approach that conceives of these equations simply as a piece of mathematics is impoverished, and unable to generate scientific explanations. On the other hand, he also warns against the dangers of trying to investigate and explain phenomena through what he describes as the distorting medium of a physical hypothesis, which can lead to a sort of blindness and rashness of conclusions (Maxwell [1855/56] 1890, p. 156). The proper methodology, according to Maxwell, is that of physical analogy, which he describes as a way of getting physical ideas without actually adopting a full physical hypothesis.

[1] For a nice history of Helmholtz's work on fluid mechanics, including his use of the electromagnetic analogy, see Darrigol (1998).

[2] There are some interesting parallels between Maxwell's and Helmholtz's methodology here and what I have called the "reciprocal correspondence principle methodology" in the context of classical and quantum mechanics. Paul Dirac, for example, frequently used this latter method to solve problems in quantum theory by first translating them into the classical context, solving them there, and then using relations such as the correspondence principle, to import the solution back into quantum theory (see Bokulich, 2008, Section 3.2). More generally, both methods illustrate an important, but often overlooked, "horizontal" dimension to model building (see, for example, Bokulich, 2003).

[3] Lydia Patton (2009) has an excellent discussion of Helmholtz's work on fluid dynamics as presaging the *Bild* (picture) theory, and its subsequent influence on Hertz and Wittgenstein.

[4] Thomson, W. (1842) "On the Uniform Motion of Heat in Homogeneous Solid Bodies, and its Connection with the Mathematical Theory of Electricity", *Cambridge Mathematical Journal* 3: 71–84.

[5] In this same letter, Maxwell also mentions to Thomson that he has been investigating hydrodynamics and gives the example of vortex motion: "I have been investigating fluid motion with reference to stability and I have got results when the motion is confined to the plane of *xy*. I do not know if the method is new. It only applies to an incompressible fluid moving in a plane" (Maxwell letter to Thomson, 15 May, 1855; Larmor, 1937, p.12). Helmholtz's treatise on vortex motion would appear just two years later.