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Abduction, tomography, and other inverse problems

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ABSTRACT

Charles S. Peirce introduced in the late 19th century the notion of abduction as inference from effects to causes, or from observational data to explanatory theories. Abductive reasoning has become a major theme in contemporary logic, philosophy of science, and artificial intelligence. This paper argues that the new growing branch of applied mathematics called inverse problems deals successfully with various kinds of abductive inference within a variety of scientific disciplines. The fundamental theorem about the inverse reconstruction of plane functions from their line integrals was proved by Johann Radon already in 1917. The practical applications of Radon's theorem and its generalizations include computerized tomography which became a routine imaging technique of diagnostic medicine in the 1970s.

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1. Peirce on abduction

Charles S. Peirce introduced in 1865 the classification of inference into deduction, induction, and hypothesis. While a typical deduction proceeds from causes to effects, hypothesis is reasoning from effects to causes. Hypothesis is thus inference to an explanation: we conclude the existence of a fact "from which, according to known laws, something observed would necessarily result" (*CP* 2.536). Frequently such inferred facts are things which "it would be impossible for us to observe directly" (*CP* 2.640).

Peirce's starting point was the observation that a paradigm example of *deduction*, the Aristotelean Barbara syllogism of the first figure, can be inverted in two different ways. Using modern logical notation and allowing singular terms, Barbara as "an explanatory syllogism" looks as follows:

(1) $(x)(Fx \rightarrow Gx)$
Fb
$\therefore Gb$.

Induction is the inference of the major premise (rule) from the minor premise (case) and the conclusion (result):

(2) Fb
Gb
$\therefore (x)(Fx \rightarrow Gx)$.

Hypothesis is the inference of the minor premise from the major premise and the conclusion:

(3) $(x)(Fx \rightarrow Gx)$
Gb
$\therefore Fb$.

(See *CP* 2.623.) Another form of Barbara involves only universal statements: for example, the supposition that light is ether waves explains why light gives certain peculiar fringes, given the law that ether waves give these fringes (Pierce, 1982, p. 267). Again, such an argument has a hypothetical inversion.

Peirce also showed how explanatory and abductive arguments can be reformulated in the context of statistical or probabilistic laws.

Later in the 1890s Peirce called hypothetical inference *abduction* and *retroduction* (see Peirce, 1992). The logical form of retroduction is like (3), but now it is natural to understand the law $(x)(Fx \rightarrow Gx)$ as a "law of succession", rather than as a "law of coexistence", stating that events of type F are necessarily or probably followed by events of type G. As causes temporally precede their effects, the inverse inference (3) from an effect to its cause involves reasoning backwards in time. Peirce's paradigm examples of retroduction included inferences about the existence of historical persons, like Napoleon Bonaparte, from the present testimony of memoirs, monuments, and public documents (*CP* 2.625; Peirce, 1998, p.

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54). Perceptual judgment is an extreme, "not fully conscious" case of abduction (*CP* 5.181). In such cases, abduction is "perfectly certain" or "compelling".

The general form of abductive inference includes cases where the tentative conclusion is an explanatory theory A (*CP* 5.189):

(4) The surprising fact C is observed;
But if A were true, C would be a matter of course.
Hence, there is reason to suspect that A is true.

According to Peirce, such abduction is conjectural, concluding that A may be true or that "there is reason to suspect that A is true". Methodologically speaking, abduction is the first stage of inquiry, leading from the data C to one of its potential explanations A, and it is followed by empirical or experimental testing of the deductive or probable consequences of the tentative hypothesis A.

The general form of abductive inference (4) may be complemented by rules or criteria for selecting the *best* explanation of data C. Peirce himself thought that abduction should be linked with considerations about the "economics of research" (Peirce, 1931–35, 1958, p. 220). Of special interest are cases where only *one* plausible explanation is available.[1]

Abduction became a hot topic in the philosophy of science in the 1960s, when N. R. Hanson proposed that Peirce's schema provides "a logic of scientific discovery" and Gilbert Harman argued that "inference to the best explanation" (IBE) is the basic form of inductive inference.[2] Today, philosophers continue discussion on the role of abduction in both scientific heuristics and justification (see Hintikka, 1998; Lipton, 1991; Niiniluoto, 1999a; Schurz, 2008). In particular, the validity of abduction is often seen as the key issue in debates concerning scientific realism (see Niiniluoto, 1999b; Psillos, 1999, 2003). Besides the work of Peirce scholars and semioticians (see Kapitan, 1997), abduction is a major theme in logic and artificial intelligence (see Aliseda, 2006), computational philosophy of science (Thagard, 1988), cognitive science, diagnostic reasoning, and model-based-reasoning (see Magnani, 2001).[3]

2. The method of analysis and synthesis

Peirce's idea of hypothetical or abductive inference, which reasons backwards from effects to causes, was not as such novel in the history of philosophy. Peirce himself adopted his term 'abduction' from the Latin translations of Aristotle's *apagoge* or 'reduction' (see Peirce, 1992).

A classical example of backward reasoning is Aristotle's discussion, in the *Nichomachean Ethics,* of the structure of deliberation: first from ends to means and then from means to ends. Aristotle knew that this double inference resembles the method of analysis and synthesis in geometry. According to the famous description of Pappus (ca. 300 AD), *theoretical analysis* goes backward from a theorem to the axioms from which it deductively follows, and *problematic analysis* goes from the desired thing to possible constructions from which it results. A successful problematic analysis tells also how many possible solutions the problem has. Synthetic reasoning then conversely gives the proof of the theorem from the axioms, or the construction of the sought thing (see Hintikka & Remes, 1974; Niiniluoto, 1999c).

The medieval followers of Aristotle called syllogistic inferences from causes to effects *compositio*, and the inverse inference from effects to their causes *resolutio*. The combination of these two inferences was called "regression" by Zabarella in the 16^{th} century. The Renaissance champions of modern science, like Galileo Galilei and Isaac Newton, followed this tradition of the "regressive method" or the "method of resolution and composition". The work of Pappus was available as Latin translation since 1566. Descartes' analytic geometry can be seen as providing an algebraic formulation of the method geometrical analysis: the sought thing is denoted by x, and the connections of x to other entities are described by an equation, which is finally solved with respect to x (see Mäenpää, 1997).

3. Inverse problems

When Peirce wrote about abduction, inverse methods had not yet been systematically studied in mathematics. His manuscripts about geometry contain only a brief remark about the "reversion and perversion" of serial orders (Peirce, 1976, p. 316).

Since the 1970s "inverse problems" have been established as a new subdiscipline of mathematics, with its own academic journals (*Inverse Problems* since 1985, *Journal of Inverse and Ill-posed Problems*, *Inverse Problems in Science and Engineering*), international congresses, scientific societies, and centres of excellence. Mathematicians working in this area have not paid attention to Peirce's concept of abduction, however. Likewise, in spite of abundant applications in various domains, the studies of abduction in logic, AI, and philosophy of science do not contain references to this new mathematical research programme.

In this section, I shall give some examples of inverse problems in connection with statistics and systems theory. In the next section, the fundamental theorem proved by Radon in 1917 is outlined and some of its important applications are described.

Simple algebraic treatments of inverse reasoning have been well known in modern mathematics. One method concerns the identification or estimation of parameters on the basis of equations. Let us assume a simple linear model between two physical variables x and y:

(5) $y = ax + b$.

Then parameters a and b can be calculated, if two pairs of values of x and y are known. Geometrically, this means that two points on a plane determine a straight line.

If (5) is a hypothetical theory, then *linear regression analysis* recommends that a and b are chosen so that they minimize the Least Square Distance of the line $y = ax + b$ from the observed data points $\langle x_1,y_1\rangle, \ldots, \langle x_n,y_n\rangle$:

(6) $\sum_{i=1}^{n} (y_i - ax_i - b)^2$.

The basic idea of this method of least squares, and its connections to observational errors, was developed by Legendre, Gauss, and Laplace already in the first years of the 19^{th} century. The statistical method of regression analysis was systematically developed around 1900 by U. Yule, Karl Pearson, and A. A. Markov.[4]

[1] Edgar Allan Poe's detective stories were composed so that they have only one backward solution (see Niiniluoto, 1999c).

[2] Carl G. Hempel analysed in 1958 the logical structure of "postdiction" or "retrodiction" (see Hempel, 1965, p. 173). For the former term, he referred to Reichenbach in 1944, for the latter term, to Ryle in 1949 and Robertson in 1895. Peirce was not mentioned at all. Similarly, Hempel's 1962 study on statistical and inductive-probabilistic explanation failed to refer to Peirce's pioneering work (see Niiniluoto, 2000). An elegant analysis of explanation, prediction, and retrodiction in discrete state systems was given by Nicholas Rescher (1970). His inspiration came from Hempel, and again no reference to Peirce's abduction or retroduction was given.

[3] A comprehensive bibliographical survey is given by Paavola (2006).

[4] Schurz (2008) shows that statistical factor analysis, a generalization of regression analysis, can be understood as a form of abduction.