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State of the field: Measuring information and confirmation

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ABSTRACT

The aim of this paper is to survey and discuss some key connections between information and confirmation within a broadly Bayesian framework. We mean to show that treating information and confirmation in a unified fashion is an intuitive and fruitful approach, fostering insights and prospects in the analysis of a variety of related notions such as belief change, partial entailment, entropy, the value of experiments, and more besides. To this end, we recapitulate established theoretical achievements, disclose a number of underlying links, and provide a few novel results.

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1. Introduction

Information is a ubiquitous term, occurring across philosophy and the sciences with a great variation in meaning. *Confirmation*, on the other hand, is a term of art in contemporary philosophy of science, defined as the impact of evidence on hypotheses. What do these two notions have in common? They both are, it turns out, central concepts when one deals with rational inference and inquiry. Loosely speaking, it seems clear that the impact of a piece of evidence (data, premise) on a given hypothesis (theory, conclusion) must reflect how the former affects an antecedent state of information concerning the latter. Relatedly, a rational agent would gather evidence because it provides information concerning certain possible states of affairs, i.e., for it can confirm/disconfirm relevant hypotheses.

The aim of this paper is to survey and discuss some key connections between information and confirmation within a broadly Bayesian framework. Indeed, a common view about information is that it is inversely related to probability (an assumption which Floridi, 2013 calls "inverse relationship principle" after Barwise, 1997, p. 491). So getting to know that the outcome of a draw from a well-shuffled deck happens not to be the seven of clubs is not very informative, for that was quite likely to be the case to begin with, and one's epistemic state would be altered only to a limited extent by this discovery. Being told that the outcome of the draw is a picture of hearts provides more information in comparison, because this singles out a small subset of possibilities that was initially rather improbable. In philosophy, this basic idea found its canonical formulation in seminal work by Bar-Hillel and Carnap (1953), who famously discussed two distinct formal representations of the information conveyed by a statement s:

$$inf_R(s) = \log[1/P(s)]$$

$$inf_D(s) = 1 - P(s)$$

The base of the logarithm is taken to be greater than unity (in the following, we will always comply with the use of $\log_2$, a fairly common choice). For the moment, subscripts "R" and "D" simply reflect the *ratio* and the *difference* involved in the corresponding expressions; but this notation will gain more relevance further on in our discussion. These classical analyses have not remained unchallenged (see, e.g., Floridi, 2004; also see Cevolani, 2013 for a neat and recent discussion), but stand as a sound basis at least for our purposes.

Mathematically, inf_R is also pivotal to so-called information theory, a well-established discipline founded by Claude Shannon (1948) with major implications in engineering and other applied

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sciences (see McKay, 2003). Ever since Bar-Hillel (1955), warnings have been flagged that conceptual confusion can potentially arise from this overlap of formalisms. In fact, in standard information-theoretic applications, $P(s)$ does *not* represent the credibility of a statement in an epistemic context, but rather the relative frequency of occurrence of a symbol in a code—a crucial difference of interpretation. However, inf_R ended up having rather wide currency in philosophy of science and related areas, too, and probably more than inf_D. A key point of this paper, anyway, is to keep both possibilities open. Thus, we will often write *inf* (with no subscript) to denote a measure of information that could be either of inf_R or inf_D.[1]

Let us now turn briefly to confirmation. A probabilistic theory of confirmation can be spelled out by a function representing the degree of support that hypothesis h receives from evidence e relative to some probability distribution P. Here, we'll rely on the background of a finite set of possible worlds, a corresponding Boolean language $\boldsymbol{L}$, and the set $\boldsymbol{P}$ of all regular probability functions that can be defined over the latter.[2] $\boldsymbol{L}_C$ will denote the subset of the consistent formulae in $\boldsymbol{L}$ (i.e., those denoting a non-empty set of possibilities). Confirmation will then be represented by some function $conf(h,e)$: $\{\boldsymbol{L}_C \times \boldsymbol{L}_C \times \boldsymbol{P}\} \to \Re$ and will have relevant probability values as its building blocks (a feature named *formality* in Tentori, Crupi, & Osherson, 2007, 2010).[3]

Note that, if an appropriate function $conf(h,e)$ is identified, a qualitative notion of confirmation can be easily derived, as follows:

(Q) *Qualitative confirmation*. A hypothesis is confirmed by some evidence just in case its complement is disconfirmed. Formally: for any $h,e \in \boldsymbol{L}_C$ (with h non-tautological) and any $P \in \boldsymbol{P}$, e confirms/is neutral for/disconfirms h if and only if $conf(h,e) \gtreqless conf(\neg h,e)$.

As for $conf(h,e)$ itself, two basic requirements will suffice for our present purposes:

(F) *Final probability*. For any fixed hypothesis, final (posterior) probability and confirmation always move in the same direction in the light of data. Formally: for any $h,e,f \in \boldsymbol{L}_C$ and any $P \in \boldsymbol{P}$, $conf(h,e) \gtreqless conf(h,f)$ if and only if $P(h|e) \gtreqless P(h|f)$.

(T) *Tautological evidence*. Any hypothesis is equally "confirmed" by empty (tautological) evidence. Formally: for any $h,k \in \boldsymbol{L}_C$ and any $P \in \boldsymbol{P}$, $conf(h,\top) = conf(k,\top)$.

We will call $conf(h,e)$ a *measure of confirmation* if and only if it satisfies both (F) and (T). As a motivation for this choice, let us first note that (F) is a virtually unchallenged principle in probabilistic theories of confirmation (see Crupi, Chater, & Tentori, 2013 for a list of references). Moreover, coupling (F) and (T) is sufficient to imply, via definition (Q), the traditional notion that, for any $h,e \in \boldsymbol{L}_C$ and any $P \in \boldsymbol{P}$, e confirms/is neutral for/disconfirms h if and only if $P(h|e) \gtreqless P(h)$.[4]

Relying on the basic points above, we now mean to present a collection of results and open issues concerning how information and confirmation are connected. Some parts of this contribution will thus draw on a background of well-known theoretical achievements, but we will often dig out material that we think is scattered or currently underappreciated in the philosophical literature, and sometimes interpolate entirely novel elements. In the next section, we will first address inference, where some given evidence and a single target hypothesis are at issue. Further on, we will be concerned more with search and inquiry, that is, with the expected value of collecting potentially relevant evidence. A final section will then outline some implications and prospects for future investigation.

2. Information, confirmation, and the impact of evidence

2.1. From information to confirmation as belief change

Being a decreasing function of $P(s)$, $inf(s)$ reflects the "unexpectedness" of s. If the evidence acquired decreases (increases) the degree of unexpectedness of a hypothesis of interest, the credibility of such hypothesis is thereby positively (negatively) affected. A simple way to convey this natural idea is to represent the *belief change* concerning h provided by e, $bc(h,e)$, by means of the plain difference between $inf(h)$ and $inf(h|e)$ (see Milne, 2014). Notably, two classical confirmation measures are thus immediately recovered[5]:

$$bc_R(h,e) = inf_R(h) - inf_R(h|e) = \log[P(h|e)/P(h)]$$

$$bc_D(h,e) = inf_D(h) - inf_D(h|e) = P(h|e) - P(h)$$

For both measures, it readily follows that $bc(h,h) = inf(h)$, an implication strongly welcomed by Milne (2012) (also see Huber, 2008 in this respect). Moreover, both measures exhibit an appealing additive behavior, in that $bc(h,e \wedge f) = inf(h)$

[1] Many authors, following Bar-Hillel and Carnap, do retain both inf_R and inf_D, either as useful theoretical constructs (e.g., Hintikka, 1968, 1970; Hintikka & Pietarinen, 1966; Kuipers, 2006; Milne, 2014; Pietarinen, 1970) or as a target of criticism (e.g., Maher, 1993, pp. 234 ff.; Levi, 1967, p. 374). When a choice is made, however, inf_R quite often prevails (see, for instance, Cox, 1961, Ch. 2; Mura, 2006, p. 196; van Rooij 2009, p. 170; Törnebohm, 1964, Ch. 3), although inf_D is found in classical works such as Hempel and Oppenheim (1948, p. 171) and Popper (1959, p. 387), and appears as a central notion—*uncertainty*—in Adams's probability logic (see, for instance, Adams, 1998, p. 31). Howson and Franklin (1985) are particularly firm in their preference for inf_R against inf_D, which they criticize as involved in Popper and Miller's (1983) famous attack on inductive probability. However, our reliance on inf_D throughout this paper does not imply in any way Popper and Miller's controversial assumption that $e \to h$ represents all of the content of h that goes beyond e (see Redhead, 1985). Indeed, logically, one can safely retain inf_D and still reject Popper and Miller's argument as unsound. (These clarifications were prompted by useful comments by an anonymous reviewer, which we gratefully acknowledge.)

[2] This set up is known to be very convenient, but has limitations. Festa (1999) and Kuipers (2000, pp. 44 ff.) discuss some important cases that are left aside here.

[3] Properly speaking, the notation should also indicate that C depends on some P in $\boldsymbol{P}$. One should write, for instance, $conf_P(h,e)$, or $conf(h,e,P)$. This amount of rigor would burden subsequent parts of our discussion inconveniently, though.

[4] *Proof.* For any $h,e \in \boldsymbol{L}_C$ and any $P \in \boldsymbol{P}$:

$P(h\|e) \gtreqless P(h)$	if and only if $P(\neg h) \gtreqless P(\neg h\|e)$	(probability calculus)
	if and only if $P(h\|e) \gtreqless P(h\|\top)$ and $P(\neg h\|\top) \gtreqless P(\neg h\|e)$	(probability calculus)
	if and only if $C_P(h,e) \gtreqless C_P(h,\top)$ and $C_P(\neg h,\top) \gtreqless C_P(\neg h,e)$	(by F)
	if and only if $C_P(h,e) \gtreqless C_P(h,\top) = C_P(\neg h,\top) \gtreqless C_P(\neg h,e)$	(by T)
	if and only if e confirms/is neutral for/disconfirms h.	(by Q)

Of course, we are dealing with the idea of confirmation as *relevance* here ("increase of firmness", in Carnap's, 1950/62 terminology: also see Good, 1968, p. 134; Salmon, 1975). In a view of confirmation as "firmness", on the other hand, $conf(h,e)$ would simply amount to an increasing function of $P(h|e)$. Interestingly, one can characterize *this* notion by replacing (T) with the following condition of *local equivalence*: for any $h,k,e \in \boldsymbol{L}_C$ and any $P \in \boldsymbol{P}$, if h and k are made logically equivalent by e, then $conf(h,e) = conf(k,e)$ (see Crupi, 2013; Crupi & Tentori, in press; Schippers, 2013).

[5] Measures ordinally equivalent to $bc_R(h,e)$ have been discussed in epistemology ever since Keynes (1921, pp. 165 ff.), while $bc_D(h,e)$ was influentially put forward by Carnap (1950/62, p. 361).