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Time and quantum theory: A history and a prospectus

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ABSTRACT

The historical part of this paper analyzes in detail how ideas and expectations regarding the role of time in quantum theory arose and evolved in the early years of quantum mechanics (from 1925 to 1927). The general theme is that expectations which seemed reasonable from the point of view of matrix mechanics and Dirac's q-number formalism became implausible in light of Dirac–Jordan transformation theory, and were dashed by von Neumann's Hilbert space formalism which came to replace it. Nonetheless, I will identify two concerns that remain relevant today, and which blunt the force of Hilgevoord's (2005) claim that the demand that time feature as an observable arose as the result of a simple conceptual error. First, I advocate the need for event time observables, which provide a temporal probability distribution for the occurrence of a particular event. Second, I claim that Dirac's use of the extended phase space to define time and (minus the) energy as conjugates is not subject to 'Pauli's Theorem,' the result that rules out time observables in von Neumann's formalism. I also claim that the need to define these event time observables leads to a novel motivation for considering Dirac's extended state space.

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1. Introduction

The conventional wisdom regarding the role of time in quantum theory is this: "time is just a parameter in quantum mechanics, and *not* an operator" (Sakurai, 1994, p. 68, original emphasis).[1] The reason for this is 'Pauli's Theorem,' a collection of results that show that (subject to a mild restriction on the Hamiltonian) conventional quantum mechanics does not permit the definition of a time observable, i.e. a self-adjoint operator canonically conjugate to energy.[2] If one wishes to have time appear as a genuine observable of the theory then this is obviously a problem, called by some "the problem of time in quantum mechanics" (Hilgevoord & Atkinson, 2011; Olkhovsky, 2011). Hilgevoord's (2005) attempted dissolution of the problem rests on his rejection of a particular motivation that one might have for wishing to regard time as a genuine observable. Hilgevoord's argument is essentially this: there is nothing problematic about time being represented by a parameter rather than an operator since *space* is represented by a parameter rather than an operator as well.

In his otherwise excellent historical survey, Hilgevoord (2005) contends that the demand that time be an observable can be traced back to a conceptual confusion common among the progenitors of quantum mechanics, in particular Dirac, Heisenberg, Schrödinger, and von Neumann. This diagnosis, and the attempted dissolution, is based on the foundational analysis presented in Hilgevoord (2002) and is repeated in his recent survey article, Hilgevoord and Atkinson (2011). There, it is summarized nicely in saying that:

> The apparent problem of time arises when [the time parameter t] is put on a par with dynamical position variables rather than the coordinates of space. The confusion has proved to be quite persistent in the quantum mechanics literature […due to the fact that] often the coordinates of space and time and the position variables of a point particle are denoted by the same symbols x, y, z (e.g. when one writes $\psi(x, y, z, t)$ for the wave function of a particle). (Hilgevoord & Atkinson, 2011, p. 650)

Hilgevoord claims that the expectation of the authors of quantum mechanics that time should be an observable was due to this confusion between space and position: guided by the role of position as an observable of the theory, they were mistakenly

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[1] Other statements of this sort include: "time is a parameter in quantum mechanics and *not* an operator" (Duncan & Janssen, 2013, p. 216,original emphasis); "Since the very beginning of quantum mechanics it is not so easy to define time at a quantum level; in the ordinary theory, in fact, it is not an observable, but an external parameter, in other words, time is *classical*." (Giannitrapani, 1997, p. 1575, original emphasis). The introduction of Aharonov & Bohm (1961) contains a similar statement, and may be the source of this concordance.

[2] See Srinivas & Vijayalakshmi (1981) for a rigorous derivation of this result.

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led to the idea that time should be observable too. When presented with an operator whose spectral values appear to correspond to points of space, it is natural to expect also an operator whose spectral values correspond to instants of time. And given the expectation of these authors that quantum mechanics would ultimately be a relativistic theory, it seemed reasonable to demand of a theory set in space–time that time and space should appear on the same footing. However, as Hilgevoord points out, the spectral values of position are *not* identical with spatial points —this correspondence is only valid for a system comprising a single particle. In general the dimension of configuration space (and so the spectrum of the position observable) is $3N$, where N is the number of particles.

Once this confusion is made apparent and it is realized that time t (a parameter) is to be contrasted with spatial coordinates x, y, z (also parameters) the apparent asymmetry is removed and so the justification for defining a time operator (i.e. a time observable) is undermined, or so Hilgevoord claims. This leads him to dismiss later developments, such as the recent use of POVMs (Positive Operator Valued Measures) to define (generalized) time observables, as conceptually confused for the same reason:

> POVMs are interesting in their own right, having many practical applications, but we shall not discuss them here, since we believe their use as a way of nullifying Pauli's objection to be fundamentally misdirected. […] there appears to be a fundamental difference between position and time in quantum mechanics. (Hilgevoord & Atkinson, 2011, p. 649)

Now, with regard to this particular justification for regarding time as an operator, I agree that Hilgevoord offers an apt diagnosis: what is being mistakenly equated here is time and position, not time and space. But while I agree wholeheartedly that it would be a mistake to confuse space, time and position in this way, I am not convinced that this was a confusion to which many (or perhaps any) of the authors of quantum theory were prone. It is my view that other reasons for defining time operators were more important to those authors—I will claim that some remain compelling today—and these are not so easily dismissed as resulting from a simple conceptual error.[3] Indeed, Hilgevoord himself advocates the construction of a quantum variable corresponding to an 'ideal quantum clock,' which fits the definition of a quantum time observable. I will argue that there is another reason to construct quantum time observables, not considered by Hilgevoord. Namely, there is a need for observables that describe the distribution of event times: outcomes of experiments that are well-localized in time as well as space.

So whereas Hilgevoord reserves his attention for time operators that can be regarded as physical clocks (that is, physical variables whose expectation value covaries with time), I advocate another class of time operators that have been considered in the quantum foundations literature: event time observables. The crux of my argument is the idea that time observables in quantum theory need not 'measure time' (as would a physical clock) but may instead serve to provide probability measures for the occurrence of events at particular (sets of) times, just as the position observable provides probability measures for the occurrence of events at particular (sets of) spatial points. This provides the means to resist Hilgevoord's accusation of pursuing a false analogy since if the event time observable concerns the location of an event in time then there is no disanalogy with the position observable, which concerns the location of an event in space (the event in question being, at first blush, something like 'the particle's being here'). There is, I claim, no relevant distinction between position in time and position in space to be drawn here.

In the historical part of this paper I will be concerned with analyzing in more detail how ideas and expectations regarding the role of time in the theory arose and evolved in the early years of quantum mechanics, from 1925 to 1927. The general theme will be that expectations which seemed reasonable from the point of view of matrix mechanics and Dirac's q-number formalism became implausible in light of Dirac–Jordan transformation theory, and were dashed by von Neumann's Hilbert space formalism which came to replace it. Nonetheless, I claim that the physical motivations behind these expectations were often sound, and in particular I will identify two aspects that remain relevant today. The first of these concerns Heisenberg's interpretation of the time–energy uncertainty principle in terms of an event: the time of a "quantum jump." The second concerns Dirac's use of extended phase space as a means to escape Pauli's Theorem.

That is, I point out Dirac's use of an 'extended' classical phase space (which includes time and energy as conjugate variables from the outset) to define his quantum dynamics means that the corresponding quantum variables are not subject to Pauli's 'no-go' theorem (nor later related results) and, moreover, his motivations for using this extended configuration space are not merely relativistic. This indicates another way to avoid this 'problem of time:' by defining an 'extended' Schrödinger equation for functions of space *and* time we can have a quantum theory in which time and (minus the) energy are represented by canonically conjugate observables, as Dirac had originally intended. I will also attempt to show how these considerations are related: exploring the first claim (that the time of an event is an observable quantity) leads naturally to the second (that quantum theory can or should be defined on extended configuration space).

The structure of the paper is as follows. In Sections 2–4 I survey three historical episodes in early quantum theory that are relevant to my claims. Section 2 is concerned with matrix mechanics and the reasons behind attempts to define time as an observable (i.e. a matrix) in that context, and their later discrediting at the hands of Pauli. Section 3 tells the story of an ill-fated (and short lived) interpretation of matrix mechanics as a theory of 'quantum jumps,' i.e. events occurring at definite times. Section 4 contains a detailed exploration of the ways that time entered into Dirac's early quantum theory, arguing that his motivations for regarding time as an operator were quite distinct from those in the matrix mechanics camp, and thus untouched by Pauli's complaints. In Section 5 I argue that the consideration of event time observables provides another legitimate source of time operators in quantum theory and survey the impact of Pauli's Theorem. Section 6 contains some details of the history of attempts to define event time observables while avoiding these 'no-go' results, ultimately as time shift covariant POVMs. I conclude by advocating a recent proposal by Brunetti, Fredenhagen, and Hoge (2010) that links the use of event time observables to Dirac's extended Schrödinger equation.

2. Time in matrix mechanics

The early years of quantum mechanics (1925–1927) were a period during which close links to classical mechanics were routinely posited and exploited, and then discarded if they conflicted with the further development of the theory. Some of these links were very suggestive of a role for time as an observable (or matrix, or q-number) of the theory. The expectation that energy and time must form a canonically conjugate pair arose within the matrix mechanics camp from the close relation of the new

[3] I do not mean to quibble with Hilgevoord's (2002) dismissal of a 'universal' time observable (what Prof. Fleming calls a 'general' time observable). Nothing I say here should be read as supporting such an idea. In fact, see Pashby (2014, Section 4.3) for my own distinct critique of this idea.