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Response to Dr. Pashby: Time operators and POVM observables in quantum mechanics ☆

Gordon N. Fleming *

Physics Department (Emeritus), Penn State University, University Park, PA 16801, USA

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ABSTRACT

I argue against a *general* time observable in quantum mechanics except for quantum gravity theory. Then I argue in support of *case specific* arrival, dwell and relative time observables with a cautionary note concerning the *broad* approach to POVM observables because of the wild proliferation available.

1. Introduction

As intended by professor Grosholz, the papers presented at the *Workshop on Cosmology and Time* have gone through revisions since the original presentation as the participants continued communicating their differing perspectives to one another. For final publication, of course, this process must be brought to a close. Consequently this response may contain some redundancies vis-à-vis Dr. Pashby's paper as well as a comment or two that do not address items remaining in Pashby's paper. For possible clarification, earlier versions of our papers can be found in the University of Pittsburgh phil-sci archive at: http://philsci-archive.pitt.edu/view/confandvol/

As in my recent papers I will follow the admonitions (which will not be defended here) of Jean Marc Levy-Leblond (1988, 1999) and Hans Christian Von Baeyer (1997), to drop the term *particle* and call the bosons and fermions of the world, *quantons*.

2. Between Pashby and Hilgevoord

Back in 1998 professor Hilgevoord (Hilgevoord, 1998), extensively referred to by Dr. Pashby (Pashby, 2013), criticized a long paper I co-authored with Jeremy Butterfield (Fleming & Butterfield, 1999), in which we discussed (among other things) Lorentz covariant 4-vector position operators, assigned to space-like hyperplanes, and with operator valued time components. Hilgevoord objected not only to the operator time components, but to the requirement of Lorentz covariance for the position operators as well! I did not then and do not now agree with these objections, for the time components were in no sense independent or general time operators, but supervened on the space components by being linear functions of them and this enabled the covariant transformation property. However, notwithstanding this episode and Pashby's fascinating account of Dirac's early explorations in building quantum mechanics (QM), I think I am sympathetic to Hilgevoord's objections (Hilgevoord, 2005) to a *general* time operator in QM. I will elaborate on this below.

On the other hand, I agree with Pashby's support of what I will call *case specific* time operators in QM, tentatively interpreted, when non-self-adjoint, as POVM observables. There are, however, delicate issues regarding the POVM interpretation of observables which I want to discuss in the context of such time operators. But first, my sympathies with Hilgevoord.

3. Time, observables and measurement

There are two brief arguments, other than Pauli's (Pauli, 1980), that I would mount against a general, canonical, time operator in QM. First, and most importantly: In QM, whether Galilean or

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* Fax: +1 814 865 0978.
E-mail addresses: gnf1@psu.edu, gnf1@earthlink.net

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Lorentz covariant, space and time or space–time, are not, themselves, dynamical systems. QM is a theory of *temporally persistent dynamical systems*, indeed of *eternal* systems, which live in a fixed classical space–time. Unlike Quantum Gravity research or Quantum Cosmology, which seek a QM *of* space–time and must have general, operator valued, space–time observables *per se*, standard QM has no such need. The basic observables of standard QM, represented by self-adjoint operators, are designed to answer questions about the possible values, or probabilities for values, of possible properties of persistent physical systems, *at specified times* (or, relativistically, on specified space-like hyperplanes). Even so-called unstable systems, which we normally think of as temporally transient, are included in this construal. We need only view the final decay products, the unstable parent quanton and the earlier formation progenitors as the final, middle and initial configurations, respectively, of a spontaneous internal transformation of *the* persistent system.

Second: I follow Ghirardi (Bassi & Ghirardi, 2003; Ghirardi, 1998; Ghirardi, Rimini, & Weber, 1986), Pearle (Pearle, 1999, 2007), Penrose (Penrose, 1986, 2000) and others in regarding primordial, stochastic state reduction (which we merely exploit in our measurements) as the really serious absentee in current QM. If and when *this* theoretical gap is filled, via improved versions of one or another of the already proposed schemes, or otherwise, I see it as only enhancing the special status of time in QM. For while state reductions (the exploited ones) can be tailored to specific observables and can have very varied relationships to spatial locations (think of reductions to near momentum eigenstates), they all occur at essentially *definite times*, either (the exploited ones) at times of our choosing or (the primordial ones) at wholly random times or, again, on space-like hyperplanes. So there would be no question of measuring *when* the primordial reductions occur and trying to measure *just when* a measurement exploited reduction occurs (within the exploiting measurement) would be an instance of measuring a case specific time observable.

This conception of the reality of apparent state reductions may, of course, be wrong and many think it so. If so, and *genuine* state reduction is replaced by an *illusion* induced by something like environmental decoherence (Schlosshauer, 2007); well, that also is an ongoing temporal process which would not, I think, alter the special status of time in QM.

The upshot is that I think Dirac, whether confused in some of his arguments (as Pashby suggests) or not, was either lucky or wise in not sticking to his original guns (Dirac, 1926) of trying to formulate QM in the extended phase space with the extended Hamiltonian satisfying a constraint equation and with time emerging as an operator. For even without gravity to deal with, and notwithstanding the invaluable contribution of Dirac's later study of constrained dynamical systems (Dirac, 1966), I suspect that such an approach to QM in general would have either encountered analogues to the kind of conceptual problems which plague quantum gravity research today or would be a trivial and inconsequential modification. In quantum gravity research the conceptual problems are real and must be faced; in the formulation of QM they would be artificial and premature. It would, of course, be interesting to have someone explore the option Dirac abandoned to see if insights lying between triviality and obscurity are to be had.

4. Time–energy indeterminacy

While we do not have a general time observable in quantum mechanics, we do have a universal time–energy indeterminacy relation (TEIR), $\Delta T\,\Delta E \geq \hbar/2$, and it is striking how exactly opposite is our interpretation of that relation from Heisenberg's early interpretation, as described by Pashby, of the original version. While Heisenberg saw ΔT as an indeterminacy in a time of occurrence and ΔE was an *interval* between precise energy values, we now have ΔE as the standard deviation indeterminacy in the system energy while ΔT is the lower bound on the *intervals* defined by $\Delta T_X = \Delta X/|\langle\dot{X}\rangle|$ for arbitrary self-adjoint observables, X. Derived by Mandelstam and Tamm (1945) from the Robertson (1929) general indeterminacy relations

$$\Delta X\,\Delta E \geq (\hbar/2)|\langle\dot{X}\rangle|, \tag{1}$$

the ΔT of their TEIR, is the time one must wait for expectation values to change by amounts comparable to the corresponding standard deviations. This immediately yields the stationarity of energy eigenstates and, as Aharonov and Bohm (1961) pointed out, it places no restriction at all on how quickly one can, in principle, perform an arbitrarily precise measurement of the energy of a physical system! However, for many states of interest, the standard deviation, ΔE, can be infinite and then (1) and the TEIR tell us nothing.

Accordingly, stronger indeterminacy relations have been derived with new time–energy relations among them (Butterfield, 2013). Uffink and Hilgevoord (1985) and Uffink (1990) have obtained one of the most interesting versions which I just mention here without further comment.

Let $\widehat{\Pi}(E)$ be the cumulative projection valued spectral resolution of the Hamiltonian, $\widehat{H} = \int E d\widehat{\Pi}(E)$. For unit norm states let $W_\alpha(\psi)$, where $0<\alpha<1$, be the size of the smallest energy interval, I, such that

$$\left\langle \psi| \int_I d\widehat{\Pi}(E)|\psi \right\rangle = \alpha. \tag{2}$$

Let, $\tau_\beta(\psi)$, where $0<\beta<1$, be the smallest time displacement such that

$$\left|\left\langle \psi \middle| \exp\left(-\frac{i}{\hbar}\widehat{H}\tau_\beta(\psi)\right) \middle| \psi \right\rangle\right| = \beta. \tag{3}$$

Then it can be shown that

$$\tau_\beta(\psi)W_\alpha(\psi) \geq 2\hbar \arccos\left(\frac{\beta+1-\alpha}{\alpha}\right). \tag{4}$$

In particular, for $\alpha=0.9$ and $\beta=\sqrt{1/2}$, one obtains, $\tau_{\sqrt{1/2}}W_{0.9} \geq 0.9\hbar$ (Hilgevoord, 1996).

5. Case specific time observables

Now I turn to case specific time observables where I agree with Pashby concerning both the possibility and the desirability of identifying and examining such observables in QM for various times of occurrence or durations.

Concepts of quantum observable times come in at least three forms: (1) times of occurrence (*arrival times*) of specified events, (2) intervals of time (*dwell times*) spent in specified regions or conditions or (3) (*relative times*) of occurrence of one event relative to a reference event. These are of a different nature from the 'property' observables for persistent systems. They acquire their objective indeterminacy from supervening on the property observables. They can be easily motivated within standard QM, beginning with the definition of case specific time operators. Until comparatively recent times such concepts have not received much attention, but are under intense examination now (Muga, Sala Mayato, & Egusquiza, 2008), and, as Pashby suggested, usually lead to non-self-adjoint operators.

Pashby mentions the very simplest (not to say simplistic) example, introduced by Aharonov and Bohm (1961), and seriously examined by Brunetti et al. (Brunetti & Fredenhagen, 2002; Brunetti, Fredenhagen, & Hoge, 2010), among Pashby's sources. This is the arrival time operator

$$\widehat{T}_0 = -\frac{1}{2}\left(\frac{m}{\widehat{p}}\widehat{x} + \widehat{x}\frac{m}{\widehat{p}}\right). \tag{5}$$