Studies in History and Philosophy of Modern Physics 52 (2015) 62–68

Contents lists available at ScienceDirect

Studies in History and Philosophy of Modern Physics

journal homepage: www.elsevier.com/locate/shpsb

Calling time on digital clocks

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ARTICLE INFO

Article history:
Received 26 October 2013
Accepted 26 March 2015
Available online 18 April 2015

Keywords:
Digital time
Clocks
Quantum mechanics
Cosmology
Foundations of physics
Perfect actions

ABSTRACT

I explore two logical possibilities for the discretization of time, termed "instantaneous" and "smeared". These are found by discretizing a continuous theory, and the resulting structure of configuration space and velocities are described. It is shown that results known in numerical methods for integration of dynamical systems preclude the existence of a system with fixed discrete time step which conserves fundamental charges universally, and a method of avoidance of this "no-go" theorem is constructed. Finally the implications of discrete time upon quantum cosmology are discussed.

When citing this paper, please use the full journal title *Studies in History and Philosophy of Modern Physics*

1. Introduction

> "The now is a link of time….for it links together past and future, since it is a beginning of one and an end of another." - Aristotle (1996a)

Whether physics can be described on a continuum or lattice is one of the oldest questions considered by philosophers in one form or another. The most famous paradox of Zeno argues against the infinite divisibility of a temporal interval – that is against continuous time (Aristotle, 1996b). Achilles is sent to chase a tortoise, which is given a head start. If we label the position of the turtle x_t for points in time t; then in each instant that Achilles reaches x_t the turtle has moved on to x_{t+1}, thus it should seem logically impossible for Achilles to catch the turtle. Viewed externally, however, one can easily verify that Achilles does indeed catch the turtle at a finite time. Of course we now know the resolution to this apparent paradox is that there can be a finite sum of an infinite number of terms, as Archimedes found. To Zeno, however, this was not known, as it was assumed that an infinite sum cannot be finite, and thus it appeared that there could not be an infinite number of time points in an interval – time should not be continuous.

Tong makes a case against a lattice reality based upon the problems with implementation in practical terms (Tong, 2011), stating "no one knows how to formulate a discrete version of the laws of physics." Furthermore he makes the compelling case that chiral fermions do not sit easily upon a lattice, and since the Standard Model is a chiral model, this means that it appears impossible to place known physics upon a lattice. Indeed lattice simulation models of chiral fermions in four dimensions seem to rely crucially upon treating the particles as living essentially on a five-dimensional lattice (Kaplan, 1992). There is an important distinction to be noted here: attempts are made to simulate the four-dimensional behavior of the particles, for which the use of extraneous mathematical structure (in this case the extra dimension) is appropriate. If, however, one were to claim that physics in fact inhabits a lattice, rather than being simulated on one, this extra structure becomes unwelcome baggage whose existence must be explained. Tong goes on to argue that the appearance of the integers in physics is constructed from an underlying continuum, an argument which mirrors the duality between a particle which exists as the excitation of a field, and a field which is observed to be composed of particles.

There are three ways in which one can respond to arguments of this type. The first, more simplistic argument is to state that what has been shown is not a "no-go" theorem against lattice constructions in four dimensions, merely that we do not yet know how to construct one. As such it is plausible that a lattice construction may be achieved in the near future, at which point all such objections would be rendered null. A second point would be that

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http://dx.doi.org/10.1016/j.shpsb.2015.03.009
1355-2198/

the problem may come about from trying to force a continuum theory onto a lattice. If discreteness is fundamental, then a continuum theory should emerge at large scales, but features of theory at the lattice level may be radically different from those of the continuum. The final method of avoiding such problems in the context of this paper is to argue merely in favor of discrete time, not space. This argument may seem unnatural on some level, as the even-handed treatment of time and space is a guiding principle for modern theories such as relativity. However it is clear that there is a physical, substantive distinction between the two at least at the level of metric signature. In practical implementations of these theories a space–time splitting is employed regularly.

In this paper I will explore the effects of introducing a discrete tick to physical systems. The paper is laid out as follows: in Section 3, I will show the effects of introducing this tick onto systems with continuous time parameter, establishing the kinematics of such treatments. This is followed by Section 4, in which I will discuss the implementation of discrete time steps in numerical simulations. Section 5 shows one practical application of these techniques to circular motion, and a way of establishing dynamics which solve some the problems found is introduced in Section 6. Finally I will note how this effects quantum gravity. But first, to clocks…

2. A note on clocks

> "They took away time, and they gave us the clock." – Abdullah Ibrahim.

The nature of physical clocks seems dichotomous at first glance. A clock is a timekeeping device, an instrument whose observation gives information used to define the interval between two events. A clock should contain a cyclic element, which describes the tick of the system. This role is performed by, for example, observations of the positions of shadows cast by the sun or the repeated dripping of water from a vessel (as was used in the earliest clocks of Egypt and Babylon) through to the oscillations of a caesium atom used in the atomic clocks of today. The clock must also be monotonic, defining unambiguously a separation of reality into past and future.

There is of course no contradiction in this. Although at first an individual system cannot be seen to be both globally monotonic and cyclic, a clock is not, in essence, a single system. Clocks consist of two distinct coupled systems, these being the cyclic and monotonic parts accordingly. The cyclic part triggers, at some point in its cycle, a distinct and discrete advance of the monotonic part, as the pendulum of a grandfather clock causes the second hand to tick upon reaching its escapement, advancing the second hand. Of course, a grandfather clock is cyclic in itself, but upon marking the end of each day, a calendar can be updated such that the overall observation of time remains monotonic.

As described thus far, the measurement of time may be refined by reducing the interval of a tick and classically there is no reason that this refinement may not, in principle, yield an arbitrary degree of accuracy. However, lurking in the small scales is the spectre of quantum mechanics and the Mandelstamm–Tamm uncertainty (Mandelshtam & Tamm, 1945) which effectively means that for any quantum clock there is an unavoidable minimum for the amount of time it takes for a wave-packet to move a distance equal to its standard deviation, for example. For a comprehensive review see Butterfield (2013). This minimum is dependent upon the physical nature of the clock, so one might suspect that it is merely a practical problem to refine the tick indefinitely. However, it is conceivable that time is fundamentally discrete, with an indivisible tick.

A prime candidate for discretization is the Planck time – the unique time that can be formed from the dimensional constants of nature (Newton's constant, Planck's constant and the speed of light). The Planck time, around 10^{-42} s, is the time interval after the big bang on which quantum gravity effects are thought to be dominant, and the time-scale on which we would expect to see quantum corrections to Einstein's equations. Of course, the Planck time only gives a broad order-of-magnitude estimate for the scale at which time could be discrete. In quantum cosmological models it is certainly true that quantum effects can be present for several multiples of the Planck time, however in most such models what this really provides is an upper bound of sorts on the discreteness scale: it is entirely possible that the discreteness could be present at orders of magnitude shorter scales than this. If the time were discrete at longer scales than the Planck time, most quantum models of gravity would require significant modifications.[1] The upshot is that the dynamics of the tick may in fact be unavailable as an observable, and thus the only reading of time one can get is that of the monotonic part, reading time as though from a digital clock.

Within Quantum Gravity, issues regarding implementation within the Hamiltonian framework are so severe that they have been dubbed "The Problem(s) of Time". Some state that this consists of as many as eight separate yet connected issues (Anderson, 2010). The purpose of this paper is not to address such issues, but I will point out that even in the symmetry reduced mini-superspace models which are used ubiquitously in quantum cosmology time evolution is measured with respect to a scalar field. If one is even-handed in treating both geometrical and matter variables, one must apply the same "polymer quantization" (Corichi, Vukasinac, & Zapata, 2007) to both, and thus the universe is imbued with a discrete tick.

The role of a clock within a physical system is split into three parts by Bush (1990a,b). First, time as measured may be "external" or "pragmatic" – there is no coupling between the dynamical system being observed and the clock used to measure time within that system. In a classical sense, external time can be said to be measuring some aspect of Newton's absolute time on which dynamics takes place. Second, an "intrinsic" time is one which is measured as some quantity of the system itself, such as the readout of a digital display, or the position of the hands on the face of a watch. Third, "observable" or "event" time is a direct measurement of some physical quantity which is taken to be time itself, such as the position of a particle. Throughout this paper I shall always have the idea of intrinsic time in mind, as an external time can be made intrinsic simply by extending the configuration space of a system with external time by taking the product of the configuration space with the configuration space of the clock. To those interested in relational observables, such as the cosmologist, intrinsic time is all one can work with – there is no external space on which a clock can live. In terms of quantum gravity, any physical clock must have a mass and thus interact gravitationally with all other components of the system through its action on space–time. Therefore cosmologically all observable time is intrinsic, which conforms with the relational program of Rovelli (1996), in which the explicit role of time is never invoked (Rovelli, 2011). A similar notion was developed by Barbour (1994, 1999).

3. Discrete time

> "God made the integers, all the rest is the work of man." – Kronecker.

[1] This argument is entirely orthogonal to those of the size of the universe at quantum gravity scales – models such as Loop Quantum Cosmology exhibit quantum gravitational effects for a duration of time determined by a modified Friedmann equation. In the case of the flat Robertson–Walker geometries, for example, the spatial extent of the universe plays no role in dynamics and may be infinite.