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The place of probability in Hilbert's axiomatization of physics, ca. 1900–1928

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ABSTRACT

Although it has become a common place to refer to the 'sixth problem' of Hilbert's (1900) Paris lecture as the starting point for modern axiomatized probability theory, his own views on probability have received comparatively little explicit attention. The central aim of this paper is to provide a detailed account of this topic in light of the central observation that the development of Hilbert's project of the axiomatization of physics went hand-in-hand with a redefinition of the status of probability theory and the meaning of probability. Where Hilbert first regarded the theory as a mathematizable physical discipline and later approached it as a 'vague' mathematical application in physics, he eventually understood probability, first, as a feature of human thought and, then, as an implicitly defined concept without a fixed physical interpretation. It thus becomes possible to suggest that Hilbert came to question, from the early 1920s on, the very possibility of achieving the goal of the axiomatization of probability as described in the 'sixth problem' of 1900.

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1. Introduction

It has become a commonplace to refer to the 'sixth problem' of David Hilbert's (1862–1943) famous Paris lecture of 1900 as the central starting point for modern axiomatized probability theory (Hochkirchen, 1999; Schafer & Vovk, 2003, 2006; Von Plato, 1994, Chapter 7).[1] Here, Hilbert proposed to treat 'by means of axioms, those physical sciences in which mathematics plays an important part; in the first rank [is] the theory of probabilities' (Hilbert, 1900 [2000], 418). The inclusion of, on the one hand, the axiomatization of *physics* among the other 22 unsolved mathematical problems and, on the other hand, the 'theory of probabilities' among the *physical theories* to be axiomatized, has long puzzled commentators. Although it has been abundantly shown, in recent years, that 'Hilbert's [lifelong] interest in physics was an integral part of his mathematical world' (Corry, 2004, 3) (e.g. Corry, 1997; Corry, 1999a, 1999b, 1999c, 1999d, 2006c) the place of probability within his own project of the axiomatization of physics has received comparatively little explicit attention.

It is the aim of this paper to provide an account of the development of Hilbert's approach to probability between 1900 – the year in which called for the axiomatization of physics in his Paris address – and 1928 – the year in which he attempted to axiomatize quantum mechanics. On the basis of the extensive primary and secondary literature available, this period can be separated into four partially over-lapping sub-periods: 1900–1905 (Section 2), 1910–1914 (Section 3), 1915–1923 (Section 4) and 1922–1928 (Section 5).[2] From the fact that each of these sub-periods corresponds to a specific position vis-à-vis, on the one hand, the foundations of physics and, on the other hand, probability theory

[1] See, for example, Browder (1976), Corry (1997), Gray (2000a), (2000b), Reid (1996, Chapter 10), Wightman (1976) and Yandell (2002) for accounts of the background and influence of Hilbert's lecture entitled 'Mathematical Problems' (Hilbert, 1900 [2000]).

[2] The first part begins with Hilbert's (1900) Paris lecture and ends with the promises for the axiomatization of several physical disciplines in his 1905 lecture course. Following the fifth chapter of Corry (2004), entitled 'From mechanical to electromagnetic reductionism: 1910–1914', the second part begins with Hilbert's lecture courses on mechanics and the kinetic theory and ends with his discussion of theories of matter and electromagnetism. The third part begins with the completion of the 'Foundations of Physics' and ends with his lectures of 1921–1923. The fourth one starts around 1922 and ends with Hilbert's joint paper on quantum mechanics of 1928. Obviously, these parts of are not separated by clean-cut boundaries – as, for instance, Corry (1999b) makes clear.

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follows the paper's main observation; namely, that there was a fundamental change in Hilbert's approach to probability in the period 1900–1928 – one which suggests that Hilbert himself eventually came to question the very possibility of achieving the goal of the mathematization of probability in the way described in the famous 'sixth problem' ('Mathematical treatment of the axioms of physics'). In brief, Hilbert understood probability, firstly, as a mathematizable and axiomatizable branch of physics (1900–1905), secondly, as a vague statistical mathematical tool for the atomistic-inspired reduction of all physical disciplines to mechanics (1910–1914), thirdly, as an unaxiomatizable theory attached to the subjective and anthropomorphic part of the fundamental laws for the electrodynamical reduction of physics (1915–1923) and, fourthly, as a physical concept associated to mechanical quantities that is to be implicitly defined through the axioms for quantum mechanics (1922–1928). Because Hilbert tended not to stress the 'state of flux, criticism, and improvement' (Corry, 2004, 332) that his deepest thoughts about physical and mathematical issues were often in there inevitably is a certain amount of speculation involved in connecting these four sub-periods. Consequently, what follows is to be considered as one possible way of accounting for Hilbert's remarkable change of mind in the period 1900–1928.

1.1. Overview of the argument

(1) Between the years 1900 and 1905, Hilbert proposed not only to extend his axiomatic treatment of geometry to the physical theory of probability, but also to let this treatment be accompanied by the further development of its (inverse) applications in mathematical-physical disciplines (Corry, 2006c). On the one hand, given that an axiomatization is to be carried out retrospectively, the suggestion that geometry was to serve as a model for the axiomatization of probability implied that Hilbert thought of the theory as a more or less well-established scientific discipline. On the other hand, the fact that the statistical 'method of mean values' for the kinetic theory of gases was to be rigorized by means of probability theory's axiomatization pointed, firstly, to the unsettled status of probabilistic methods in physics and, secondly, to the possibility of having the axiomatic method restore it. (2) The years 1910–1914 could be separated into three phases. Firstly, from 1910–1912/1913, Hilbert explicitly elaborated the atomistic hypothesis as a possible ground for a reductionist mechanical foundation for the whole of physics in the context of several physical topics based on it (Corry, 1997, 1998, 1999d, 2000, 2004, 2006a). It was under the influence of his increasing acknowledgment of the disturbing role of probabilistic methods (e.g. averaging) in the mathematical difficulties involved in the axiomatization of 'physics in general and [the] kinetic theory in particular (Corry, 2004, 239) from the atomistic hypothesis that Hilbert, secondly, became more and more interested (from late-1911/early-1912 on) in coming to terms with these difficulties via an investigation into the structure of matter (Corry, 1999a, 1999d, 2010). Thirdly, as a result of his consideration of the mathematical foundations of physics, in the sense of radiation and molecular theory as going beyond the kinetic model 'as far as its degree of mathematical sophisticated and exactitude is concerned (Corry, 2004, 237), Hilbert eventually came to uphold Mie's electrodynamical theory of matter by late-1913/early-1914 (Corry, 1999b; Mehra, 1973; Section 3.4, see also Battimelli, 2005; McCormmach, 1970). (3) The third period (1915–1923) pivots around the appearance, in 1915, of the 'Foundations of Physics' in which Hilbert presented a unified field theory, based on an electrodynamical reductionism, that combined Mie's theory and Einstein's (non-covariant) theory of gravitation and general relativity (e.g. Corry, 2004; Majer & Sauer, 2005; Renn & Stachel, 1999; Sauer, 1999, 2005; Stachel, 1999, see also Earman & Glymour, 1978; Vizgin, 2001). Hilbert's philosophical reflections, of the early 1920s, on his theory dealt with the epistemological implications of general covariance such as time-reversal invariance and new conditions for the objectivity and completeness of physical theories based on general relativity and quantum mechanics. Where probability was here accepted as a subjective 'accessorial' principle implied in the application of the laws of the new modern physics to nature, (4) in his later contributions to the axiomatization of quantum mechanics empirical probabilities were implicitly defined through the axioms of a yet uninterpreted formalism after physical requirements had been put upon them (e.g. Lacki, 2000).

2. First period. The axiomatization of probability as a physical discipline: 1900–1905

Hilbert's *Grundlagen der Geometrie* of 1899 resulted from his attempt to lay down a simple and complete system of independent axioms for the undefined objects 'points', 'lines' and 'planes' that establish the mutual relations that these objects are to satisfy (Hilbert, 1899 [1902], see also Hilbert, 1891 [2004], 1894 [2004], Toepell, 1986b). In the lecture notes to a course on the 'Foundations of Geometry' of 1894, Hilbert defined the task of the application of the axiomatic method to geometry as one of determining

> 'the necessary, sufficient, and mutually independent conditions that must be postulated for a *system of things*, in order that any of their properties correspond to a *geometrical fact* and, conversely, in order that a complete description and arrangement of all the geometrical facts be possible by means of this system of things' (Hilbert, 1894 [3] quoted in Toepell, 1986a, 58–59, my emphasis).

Hilbert was of the opinion that his axiomatization of elementary geometry was part of a more general program of axiomatization for all of natural science (e.g. Majer, 1995) and that geometry, as the science of the properties of space, must be considered as 'the most perfect of the natural sciences'[4] see Toepell, 1986a, vii, see also Corry, 2006b; Majer, 2001). The fact that Hilbert's axioms for geometry were chosen so as to reflect spatial intuition not only indicates that the axiomatic method itself is a tool for the *retrospective*, or post-hoc, investigation of the logical structure of ''concrete, well-established and elaborated [. . .] entities' (Corry, 2004, 99). But it also suggests that the difference between geometry and, for example, mechanics pertained solely to the historical stage of the development of both sciences. Where the basic facts of geometry 'are so irrefutably and so generally acknowledged [that] no further proof of them is deemed necessary [and] all that is needed is to derive [the] foundations from a minimal set of [. . .] axioms' (Hilbert, 1898–1899 quoted in Corry, 2004, 90)[5], in mechanics

> 'all physicists recognize its most *basic facts* [but] the arrangement [is] still subject to a change in perception [and] therefore mechanics cannot yet be [turned into] a pure mathematical discipline, at least to the same extent that geometry is' (Hilbert, 1898–1899 quoted in Corry, 2004, 90).

[3] In 1894 Hilbert gave a lecture course on 'The Foundations of Geometry'.

[4] Hilbert made this remark at several occassions between 1893 and 1927 - for example in his 1898/1899 lecture entitled 'The Foundations of Euclidean Geometry'. The only place where Hilbert did not stress this aspect of his position was in his *Grundlagen der Geometrie*.

[5] In the year 1898-1899 Hilbert gave his first full course on a physical topic, namely mechanics.