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The origins of Schwinger's Euclidean Green's functions

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ABSTRACT

This paper places Julian Schwinger's development of the Euclidean Green's function formalism for quantum field theory in historical context. It traces the techniques employed in the formalism back to Schwinger's work on waveguides during World War II, and his subsequent formulation of the Minkowski space Green's function formalism for quantum field theory in 1951. Particular attention is dedicated to understanding Schwinger's physical motivation for pursuing the Euclidean extension of this formalism in 1958.

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1. Introduction

Schwinger's introduction of Green's function formalism for characterizing quantum field theories constitutes one of the most influential of his numerous contributions to the physics of elementary particles. He originally developed his Green's function method for waveguide problems during World War II, and then exported it into quantum field theory in a series of papers in 1951. His Minkowski space Green's functions allowed for a more general characterization of the theory than had previously been possible using perturbative techniques. In 1958 Schwinger published *On the Euclidean Structure of Relativistic Field Theory*, in which he introduced a technique for characterizing quantum field theories in Euclidean space rather than in Minkowski space.[1] Both Mehra and Milton[2] and Schweber[3] have provided significant insight into the connection between Schwinger's war work and his introduction of the Minkowski space Green's function formalism. In a retrospective lecture delivered late in his life Schwinger explicitly acknowledged this connection and showed how Green's function methods influenced his work throughout his career (Schwinger, 1993). While Schwinger is also widely credited for producing the first Euclidean formalism for field theory, the historical literature has neglected his motivations for introducing this extension of the formalism. My aim in this paper is to articulate a more complete account of this development.

Schwinger's previous work on Green's functions uniquely prepared him to make this contribution at the technical level.[4] The motivation for establishing the Euclidean extension also contains novel physical reasoning. Aspects of Schwinger's motivation can be inferred from the publication in which he introduced the formalism. However, Schwinger had considered transformations to Euclidean space in several contexts before using them as the basis for a novel formulation of field theory, and these provide further insight into his reasoning about the role of imaginary time transformations. In this paper I provide evidence that one of these contexts, a talk he delivered in 1957 on dispersion relations to determine the structure of Green's functions, contains the earliest articulation of the central physical insight that motivated the development of a formulation of quantum field theory in Euclidean space.

Schwinger was certainly not the first to transform field theoretic quantities into Euclidean space. Both Dyson and Wick had transformed to Euclidean space during calculations before him.

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[1] Schwinger (1958b).
[2] Mehra & Milton (2000).
[3] Schweber (1994).

[4] Late in his life Schwinger acknowledged this when he claimed that "… although it could have appeared any time after 1951, it was 1958 when I published *The Euclidean Structure of Relativistic Field Theory*" (Schwinger, 1993, p. 7). This is a reference to the fact that the 1951 Green's functions papers provided the technical framework for the 1958 Euclidean Green's function paper.

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Moreover, during the period in which Schwinger was developing Green's function formalism, mathematical physicists working in the axiomatic approach to field theory had also become interested in rigorously determining the region of analyticity of Green's functions. In fact, an influential paper of Wightman's demonstrates that the analyticity region of Green's functions includes the Euclidean region as a special case.[5] The novel aspect of Schwinger's contribution was to provide a complete formalism for field theory which emphasized the importance of the Euclidean region in particular. The characterization of quantum field theories in terms of Euclidean Green's functions, which have come to be known as Schwinger functions, is an important technique for the development of constructive models of the theory. Of the few rigorous models of field theory that have been constructed, many have existence proofs which rely critically on the Schwinger functions. The Euclidean formalism is also the basis for the enormously productive analogy between quantum field theory and statistical mechanics. Despite its eventual importance, the initial reception of the Euclidean formalism was not enthusiastic. Schwinger's characterization of the theory in terms of Minkowski space Green's functions was already widely viewed as abstract and overly formal in comparison to Feynman's graphical techniques. Moreover, the move from an underlying manifold of Minkowski space to a Euclidean space was unintuitive, given that what was being represented in the theory was fields in Minkowski space. For these reasons, Schwinger's motivation for developing the Euclidean formalism calls for explanation.

Understanding the origins of Schwinger's Euclidean Green's functions requires not only an understanding of how he developed the technical apparatus used to capture the formalism, but also his physical motivation for extending it to Euclidean space. The following section discusses the work of Mehra and Milton, and Schweber, on the developments which led to Schwinger's Minkowski space Green's function formalism for quantum field theory. In particular, it includes discussion of Schwinger's novel application of Green's function methods during World War II, and how he exported the technique into quantum field theory in a series of 1951 papers. In the third section I identify aspects of the motivation that led him to generalize the formalism developed in 1951 to the Euclidean formulation in his work on the CPT theorem and in a talk on dispersion relations delivered in 1957. I present evidence that this was the first context in which Schwinger articulated a critical piece of the motivation for developing the Euclidean formalism. In the fifth section I discuss Schwinger's introduction of the Euclidean formalism in 1958, and connect it to the broader context discussed in the third section. I conclude by reviewing the argument and connecting Schwinger's contribution to two latter developments.

2. Green's functions for waveguides and field theory

The development of quantum field theory stalled during World War II as many theoretical physicists left their research positions to work on projects for the war. A small group of theorists spent the war working on radar technology at the MIT Radiation Laboratory. Hans Bethe produced the initial work on the project and, in 1942, invited several other physicists including Schwinger to collaborate with him. As a graduate student at Columbia, Schwinger's earliest work focused primarily on phenomenological problems in nuclear physics. He had already obtained his first professorship at Purdue when Bethe recruited him to come to the Radiation Laboratory. Before going to MIT, Schwinger was invited to Los Alamos by Oppenheimer. It would have been a natural position for him as much of his early work had been dedicated to nuclear physics.[6] However, Schwinger declined Oppenheimer's offer and spent the years of the war working on waveguide problems for radar equipment at MIT. Schwinger's decision to go to the Radiation Lab instead of Los Alamos had an important effect on the trajectory of his research. His work on waveguide problems influenced his work on field theory after the war in a number of important ways. The applied physics and engineering problems that Schwinger solved at the Radiation Lab were the origin of a calculational technique that he directly imported into quantum field theory after the war. This section briefly explains how Schwinger's work at the Radiation Lab led to the development of his modern use of Green's functions in field theory. Detailed treatments of these developments can be found in work by Mehra and Milton[7] and Schweber.[8,9]

Before the war the Radiation Lab was staffed almost exclusively with electrical engineers. The directors of the lab looked to theoretical physicists with the onset of the war because a more complete theoretical understanding of the systems being used could save time in the development process. Unlike the systems that the engineers were accustomed to, microwave radio devices had a size on the order of the wavelength of the radiation they produced. Because of this they had to be engineered to transfer energy through metallic waveguides rather than wires. Understanding the properties of such systems required dealing directly with the electromagnetic fields rather than currents and voltages. Solving Maxwell's equations for the fields was complicated by the fact that realistic applications contained many different obstacles to the propagation of the radiation through the waveguides.[10]

Schwinger's task at the Radiation Lab was to develop a framework for understanding the propagation of the radiation through complex geometries involving many obstacles. The method Schwinger developed was based centrally on the use of Green's functions to describe the propagation of the modes in the radiation. Green's theorem provides a connection between volume integrals and surface integrals over volumes. Even before Schwinger's work on waveguides it was commonplace to use Green's functions to solve electrodynamic problems. Schwinger's method was different in that he treated Green's functions as functional operators that define a linear relation between a field inside a region and the boundary conditions for the field on the surface around that region. According to Mehra and Milton, the oldest record of this approach is contained in the 1943 MIT Radiation Laboratory Report 43–44 of which Schwinger was the sole author.[11] He modeled the systems with a modified form of Maxwell's equations.[12] To solve these equations Schwinger defined an electric field Green's function in terms of which the

[5] Wightman (1956).

[6] In an interview with Schweber he explained that "I would like to think that I had a gut reaction against [going]. I was probably the only active theoretical nuclear physicist who wasn't there. There must have been some deep instinct to stay" (Schweber, 1994, p. 295).

[7] Mehra & Milton (2000).

[8] Schweber (1994, 2005).

[9] Though it will not be discussed further in this paper, it is worth noting that his work at the Radiation Lab seems to have influenced his perspective on the nature of physical theorizing in a way that informed his unique perspective on renormalization theory. For further discussion of this connection see Kaiser (2009, pp. 41–42), Galison (1997, pp. 820–827), and Schwinger (1983). This perspective lead him to reject operator field theory for his own source theory later in his career. This aspect of Schwinger's thinking has been discussed in Cao (1998), Cao & Schweber (1993), Mehra & Milton (2000), and Mehra, Milton, & Cao (2003).

[10] For further discussion of the general theoretical project at the Radiation Lab see Mehra & Milton (2000, pp. 105–106). For a comprehensive account see Brown (1999).

[11] Mehra & Milton (2000, p. 119).

[12] Levine & Schwinger (1950).