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Four tails problems for dynamical collapse theories

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ABSTRACT

The primary quantum mechanical equation of motion entails that measurements typically do not have determinate outcomes, but result in superpositions of all possible outcomes. Dynamical collapse theories (e.g. GRW) supplement this equation with a stochastic Gaussian collapse function, intended to collapse the superposition of outcomes into one outcome. But the Gaussian collapses are imperfect in a way that leaves the superpositions intact. This is the tails problem. There are several ways of making this problem more precise. But many authors dismiss the problem without considering the more severe formulations. Here I distinguish four distinct tails problems. The first (bare tails problem) and second (structured tails problem) exist in the literature. I argue that while the first is a pseudo-problem, the second has not been adequately addressed. The third (multiverse tails problem) reformulates the second to account for recently discovered dynamical consequences of collapse. Finally the fourth (tails problem dilemma) shows that solving the third by replacing the Gaussian with a non-Gaussian collapse function introduces new conflict with relativity theory.

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1. Dynamical collapse theories and the problem of outcomes

Quantum mechanics suffers from the measurement problem. There are several ways of stating this problem. A useful formulation is the *problem of outcomes* (Maudlin, 1995). Two propositions made plausible by quantum mechanics are prima facie incompatible with a third independently plausible proposition:

(A) The wave-function of a system is complete in the sense that the wave-function specifies (directly or indirectly) all of the physical properties of a system.
(B) The wave-function always evolves in accord with a linear dynamical equation (e.g. the Schrödinger equation).
(C) Measurements always (or at least usually) have single determinate outcomes i.e. at the end of the measurement the measuring device indicates a definite physical state.

Propositions (A)–(C) cannot all be true at once. To illustrate: let the wave-function of a (macroscopic) measuring device d be described by $|\text{ready}>_d$ meaning that d is ready to measure the physical state of some particle. Let $|0>$ and $|1>$ describe the distinct values of some two-valued property (e.g., spin); d is designed to detect which of these two states a particle is in. If the wave-function for d and particle p is initially $|\text{ready}>_d|0>_p$ then switching d on (i.e. performing the measurement) gives $|\text{‘0’}>_d|0>_p$ where $|\text{‘0’}>_d$ means that d has detected that p is in state $|0>$ (and displays this e.g. using a pointer). Similarly, if the initial wave-function is instead $|\text{ready}>_d|1>_p$ then turning the device on gives $|\text{‘1’}>_d|1>_p$. Now let the initial wave-function of p be the linear superposition $a|0>_p+b|1>_p$ such that a and b are nonzero and $|a|^2+|b|^2=1$. Then the complete initial wave-function is as follows:

$$|\text{ready}>_d(a|0>_p+b|1>_p) \quad (1)$$

The linearity of the dynamics (B) guarantees that if d is switched on then the microscopic superposition will be magnified up into the macroscopic object yielding an entangled superposition:

$$a|'0'>_d|0>_p+b|'1'>_d|1>_p \quad (2)$$

Since d only displays three possible states (ready, ‘0’ and ‘1’), (2) does not represent a single definite measurement outcome. Thus,

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if propositions (A) and (B) are true then (given the kinds of physical states that actually obtain) proposition (C) cannot be true. But (C) is extremely plausible: it is apparently confirmed by observations of the post-measurement states of measuring devices. This is the problem of outcomes.

Solutions can be categorised in terms of which proposition they reject. Additional variables theories (e.g., Bohm (1952)) reject (A): the wave-function is incomplete and definite measurement outcomes are determined by additional variables. Orthodox textbook quantum mechanics denies (B): when measurement occurs the linear dynamics abruptly stops governing the measured system and the system's wave-function collapses into a definite (non-superposition) state. The probability that the wave-function collapses into one of its component states is given by the absolute value squared (*mod-square*) of that component's coefficient. The inadequacy of appealing to the vague notion of measurement is the traditional formulation of the measurement problem (Albert, 1992: chapter 4).

The theories at issue – dynamical collapse theories – deny (B). Moreover, they solve the measurement problem by formulating the collapse process precisely, without reference to "measurement" or cognates. I will illustrate dynamical collapse theories using the original GRW spontaneous collapse theory (Ghirardi, Rimini & Weber, 1986) and the modern matter-density theory (Ghirardi, Grassi, & Benatti, 1995). The latter denies (A) as well as (B), supplementing the wave-function with a matter-density distribution.[2]

But first it will be worth considering the Everett (or "many-worlds") interpretation,[3] which denies (C): the wave-function is complete and evolves only in accord with the linear dynamics. Expressions (1) and (2) are interpreted as a microscopic superposition causing a macroscopic system (d) to bifurcate into two distinct devices that report the two possible measurement outcomes. This does not contradict our experience because human observer h who is initially ready to observe d's result ($|\text{ready}>_h$) will in accord with the linear dynamics branch too:

$$a|'0'>_h|'0'>_d|0>_p + b|'1'>_h|'1'>_d|1>_p \tag{3}$$

The two separate terms in this superposition will (in realistic scenarios) undergo decoherence thereby evolving independently and will thus represent distinct macroscopic worlds within a single quantum mechanical multiverse.

Many-worlds theory is worth mentioning here because, as we shall see, the tails problem (when properly formulated) implies that the formalism of dynamical collapse theories describes a multiverse of some sort. This undermines collapse theories since their goal is to retain and explain (C) but (C) is inconsistent with many-worlds theory.

2. The GRW theory

The idea behind the original GRW theory is simple: elementary particles have a tiny probability per unit time for spontaneously collapsing into a definite position. Measuring devices are composed of many entangled particles and so have an extremely high probability per unit time for collapsing into a definite position. In other words, although one isolated particle rarely spontaneously collapses, one non-isolated particle will certainly collapse if a particle it is entangled with collapses. GRW postulate that particles have a 10^{-16} probability per second for spontaneously collapsing.[4] Systems composed of 10^{23} entangled particles will then collapse around 10^7 times per second. The probability that a given wave-function will collapse onto one of its components is given by the mod-square of that component's coefficient. This is how GRW recover the probabilistic predictions of textbook quantum mechanics. Note the two distinct roles played by probability: there is the probability per unit time for spontaneous collapse and there is the probability that the collapse will be centred on a given wave-function component.

It will be useful to distinguish the idealised GRW theory from the realistic GRW theory. Consider the following wave-function of a particle confined to the x-axis:

$$a_1|x_1> + a_2|x_2> + a_3|x_3> + \ldots + a_n|x_n> \tag{4}$$

In the idealised theory this particle has a 10^{-16} probability per second for spontaneously collapsing into one of the components. The probability that the particle collapses to component $|x_i>$ is $|a_i|^2$. Now consider the transition from (1) to (2). In the idealised theory (2) is unstable and will immediately collapse into $|'0'>_d|0>_p$ with probability $|a|^2$ or $|'1'>_d|1>_p$ with probability $|b|^2$. This guarantees a definite measurement outcome.

Collapse in the idealised theory reduces all but one of the coefficients to zero, while the mod-square of the chosen coefficient goes to one. But this collapse function is unphysical.[5] This is due to position/momentum incompatibility. The more confined the position wave-function the more spread out the momentum wave-function. The more spread out the momentum wave-function the more equiprobable all possible states of momentum become. The relationship between energy and momentum then yields drastic post-collapse violations of energy conservation: ones that we know by experiment do not occur. So GRW formulated the collapse function as a Gaussian. The collapse then raises the mod-square of the chosen coefficient – the collapse centre – close to one while reducing the mod-square of all other coefficients close to zero but never actually to zero. GRW carefully chose the probability per unit time for spontaneous collapse and the width of the bell curve of the Gaussian (10^{-5} m), so that the energy conservation violations are consistent with known experiments.[6]

But in formulating the collapse function consistently with experiments, GRW may have undermined the theory's ability to explain definite measurement outcomes. We must now redefine what is meant by collapse. Collapsing into a wave-function component now means something like "shifting *most* of the mod-square value to that component". Reconsidering the transition from (1) to (2): (2) is unstable but there will be no transition to a state represented by one of the components as in the idealised theory. Rather, the post-collapse state is as follows:

$$c|'0'>_d|0>_p + d|'1'>_d|1>_p \tag{5}$$

where c and d are the nonzero, $|c|^2+|d|^2=1$, and either $|c|^2 \gg |d|^2$ or $|d|^2 \gg |c|^2$. The probability that $|c|^2 \gg |d|^2$ is $|a|^2$ while the probability that $|d|^2 \gg |c|^2$ is $|b|^2$. To analyse the adequacy of this theory we must ask whether (5) ultimately makes sense as a description of a definite measurement outcome.

This theory is often denoted GRW_0 to distinguish it from GRW_M (the matter-density formulation).[7] GRW_M defines a three-dimensional

[2] Ghirardi et. al. (1995: section 3) motivate the matter-density addition as a solution to the tails problem (see Section 5.1 below). Meanwhile Allori, Goldstein, Tumulka, and Zanghi (2008: section 4.3) motivate the addition as a solution to the problem of the high (3 N) dimensionality of the wave-function. Whether the dimensionality problem undermines (A) is controversial (Albert (2013)).

[3] Everett (1957), Saunders, Barrett, Kent, and Wallace (2010), Wallace (2012).

[4] In more sophisticated variants this holds for nucleons while electrons collapse more infrequently (Pearle and Squires, 1994).

[5] Even apart from the normalisation problem for delta functions.

[6] For discussion see Feldman and Tumulka (2012).

[7] There is also GRW_F (the flash ontology) on which matter is composed of discrete space-time points called "flashes". There is one flash for each collapse. The position of the flash is the position of the collapse centre and the time of the flash is the time of the collapse. See Bell (1987: chapter 22) and Allori et. al. (2008: section 3.2).