Studies in History and Philosophy of Modern Physics 49 (2015) 57–72

Contents lists available at ScienceDirect

Studies in History and Philosophy of Modern Physics

journal homepage: www.elsevier.com/locate/shpsb

Outline of a dynamical inferential conception of the application of mathematics

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ARTICLE INFO

Article history:
Received 29 July 2014
Received in revised form
7 October 2014
Accepted 12 January 2015
Available online 10 February 2015

Keywords:
Applicability of Mathematics
Genesis of general relativity
Absolute differential calculus
Inferential conception
Einstein
Grossmann

ABSTRACT

We outline a framework for analyzing episodes from the history of science in which the application of mathematics plays a constitutive role in the conceptual development of empirical sciences. Our starting point is the inferential conception of the application of mathematics, recently advanced by Bueno and Colyvan (2011). We identify and discuss some systematic problems of this approach. We propose refinements of the inferential conception based on theoretical considerations and on the basis of a historical case study. We demonstrate the usefulness of the refined, dynamical inferential conception using the well-researched example of the genesis of general relativity. Specifically, we look at the collaboration of the physicist Einstein and the mathematician Grossmann in the years 1912–1913, which resulted in the jointly published “Outline of a Generalized Theory of Relativity and a Theory of Gravitation,” a precursor theory of the final theory of general relativity. In this episode, independently developed mathematical theories, the theory of differential invariants and the absolute differential calculus, were applied in the process of finding a relativistic theory of gravitation. The dynamical inferential conception not only provides a natural framework to describe and analyze this episode, but it also generates new questions and insights. We comment on the mathematical tradition on which Grossmann drew, and on his own contributions to mathematical theorizing. The dynamical inferential conception allows us to identify both the role of heuristics and of mathematical resources as well as the systematic role of problems and mistakes in the reconstruction of episodes of conceptual innovation and theory change.

When citing this paper, please use the full journal title *Studies in History and Philosophy of Modern Physics*

1. Introduction

The goal of this paper is to gain a better understanding of the interaction of mathematics and physics in the genesis of empirical theories, and to contribute to the philosophical debate of the application of mathematics in empirical science. We intend to develop further a framework for thinking systematically about the application of mathematics. We will apply this framework to an important historical case, the use of the “absolute differential calculus”, what is now called tensor calculus, in the genesis of general relativity.[1]

The problem of understanding how mathematics is applied in the empirical sciences has been discussed in the philosophy of mathematics, but until recently, the debate has only been marginal in comparison to the more dominant discussion of problems associated with pure mathematics.[2] One of the starting points of the debate, some 50 years ago, is a famous paper by Wigner (1960) on the “unreasonable effectiveness of mathematics in the natural sciences.” Wigner formulated his astonishment in general terms, and his examples were taken from a variety of different instances of applications from different epochs. But, historically, we believe that the background for Wigner's paper, and its impact, was the effectiveness that mathematics had borne out in the first half of

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[1] The present paper draws on Räz (2013, Part II).

[2] For recent survey articles on the issue of applicability, see Colyvan (2009) and Steiner (2005).

http://dx.doi.org/10.1016/j.shpsb.2015.01.004

the twentieth century with the emergence of both the general theory of relativity and the development of quantum theory.

Although Wigner recognized the generality of the problem, in earlier times the problem of applicability did not arise as from a moment of surprise. Euclidean geometry deals with the geometry of straight lines and circles, and solves construction tasks only with the help of ruler and compass. To be sure, it gives an axiomatic treatment of the geometry of three-dimensional ruler–compass space and it proves its assertions using language and symbolism. But the origin of its theorems in practical geometric experience, and the naturalness of its axioms for our physical world were all too obvious. When Hilbert stripped the geometric axioms of their direct meaning, he still insisted that historically, geometry was a natural science. It had only evolved to a state where its concepts and results had become so firm that no one doubted their validity any more, and they could be entirely transformed into a field of mathematics.

The origins and applicability of differential calculus may be a similar case. Conceived by its creators as a general tool to describe physical motion—see Newton's term of "fluxion" for the (time-) derivative—it was meant to be a way of putting physical phenomena into a more rigid, practical, and effective mathematical representation. As argued by Grattan-Guinness (2008), a historical perspective helps us to alleviate much of the "unreasonableness" of Wigner's "unreasonable effectiveness."

Euclidean geometry, in all its subtlety, was for a long time never applied to anything else than physical geometry. This changed with Hilbert's understanding of axiomatics, and Hilbert himself used both the electrochemical series and the laws of heredity of drosophila as models for the Euclidean axioms of linear congruence; see Sauer and Majer (2009, pp. 420–423) and Hilbert (1930). More obvious was the versatility and generality of differential calculus, which proved to be not only applicable, but indispensable for almost any field of the natural sciences throughout the nineteenth century. But the Göttingen praise of the "preestablished harmony" between mathematics and physics was most pointedly illustrated by the example of general relativity.[3]

Steiner (1998) takes the issue of applicability a step further. He argues that there is not one problem of applicability, but many problems that have to be kept separate. Steiner maintains, *contra* Wigner, that the use of individual mathematical *concepts* in empirical science may not be the main puzzle. He emphasizes the philosophical problem that mathematics as a whole turns out to be so enormously successful, despite the fact that mathematics obeys anthropocentric criteria such as beauty and convenience.[4]

Recent discussions of applicability have shifted their focus from the discussion of problems of applicability, such as the unreasonable effectiveness, to providing a positive account of the various roles that mathematics plays in application. In the present paper, we intend to take up one of these accounts, the so-called "inferential conception" of the application of mathematics proposed by Bueno and Colyvan (2011). We find the basic outline of the inferential conception to be promising for our goal of a philosophical understanding of the application of mathematics. However, a confrontation with historical case studies forces us to extend this account. We will outline the inferential conception and its extension to a "dynamical inferential conception" in Section 2.

We will then explore the approach with an important case of the application of mathematics, an episode from the genesis of general relativity, in Section 3. This case not only has the advantage that we can draw on detailed historical analyses for our study,[5] but it is also an example from the history of science where mathematics has played a prominent role in concept formation. More specific reasons for choosing this particular episode will be given in Section 3.3

We do not claim that this case is particularly typical, or that it warrants general lessons about how mathematics is applied. It merely serves as a first case that exhibits some important characteristics of application. It will be necessary to extend the examination of the use of mathematics to other episodes in the genesis of general relativity, and to other cases, such as the discovery of Maxwell's equations or the history of quantum mechanics, and finally to other fields of empirical science, in order to gain general and stable insight into this issue. In short, the philosophical account will serve as a conceptual framework, which will help us to understand the case better; this enhanced understanding, in turn, will lead to a refinement of the account.

A historical approach to the issue of application and applicability has itself a historical tradition, and there are systematic reasons why the examination of historical cases is particularly fruitful. A historical approach has been part of the discussion at least since Steiner (1998) formulated one problem of applicability as a puzzle about the surprising role of mathematics in discovery. The inferential conception, as proposed by Bueno and Colyvan (2011), has been designed to capture the historical process of application. Systematically, we are not only interested in the finished product of the process of applying mathematics, but we are also interested in the process itself. We are convinced that in order to fully appreciate a mathematically formulated empirical theory, it is indispensable to understand the historical process that led to this theory. The process of applying mathematics to empirical problems plays an important role in the formation of scientific concepts, and, more generally, in theory dynamics.

2. The dynamical inferential conception

In this section, we introduce a philosophical framework for the process of the application of mathematics. We begin by sketching two existing accounts of application, the so-called "mapping account" by Pincock (2004) and the "inferential conception" proposed by Bueno and Colyvan (2011), a more flexible version of the mapping account. We then discuss certain problems of the inferential conception. This motivates an extension of the approach to what we call the "dynamical inferential conception."

2.1. The mapping account and the inferential conception

Bueno and Colyvan (2011) use a familiar picture of applying mathematics as a foil for their own account. On this picture,

[3] In a talk held in Copenhagen in March 1921, Hilbert said: "The mathematician, who has noticed with surprise so often already the preestablished harmony between his own thinking and the world, is almost forced to the conception that nature had purposely been created in such a way that in order to grasp her the deepest mathematical speculations are needed." (Der Mathematiker aber, der schon so oft die praestabilirte Harmonie zwischen seinem Denken und der Wirklichkeit mit Staunen bemerkt, wird fast zu der Vorstellung gezwungen, als sei die Natur eigens so eingerichtet, dass es zu ihrer Erfassung der tiefsten mathematischen Spekulationen bedarf.) (Sauer & Majer, 2009, p. 387) In his lectures on the development of mathematics in the nineteenth century, Felix Klein wrote: "But the wonderful harmony, which we find between the developments of the pure mathematicians and the intellectual constructions of the theoretical physicists, is confirmed once again in an extended realm." ("Die wunderbare Harmonie aber, welche zwischen den Entwicklungen der reinen Mathematiker und den Gedankenkonstruktionen der neueren Physiker besteht, bewährt sich aufs neue auf einem erweitertem Gebiete.") (Klein, 1927, p. 79), see also his comments on Riemannian geometry and general relativity in Klein (1921, pp. 557–558).

[4] Steiner discusses the discovery of the field equations of GR, one of our case studies, on pp. 94. We will turn to his argument in Section 3.3.2.

[5] The literature on the genesis and history of general relativity is quite extensive. A good starting point is the four volumes on the genesis of general relativity edited by Jürgen Renn (2007). More specific items will be cited where pertinent.