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Mesh and measure in early general relativity

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ABSTRACT

In the early days of general relativity, several of Einstein's readers misunderstood the role of coordinates or "mesh-system" in ways that threatened the basic predictions of the theory. This confusion largely derived from intrinsic defects of Einstein's first systematic exposition of his theory. A few of Einstein's followers, including Arthur Eddington, Hermann Weyl, and Max von Laue, identified the interpretive difficulties and solved them by combining a deeply geometrical understanding of the theory with detailed attention to the concrete conditions of measurement.

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> There is no fundamental mesh-system. In particular problems, and more particularly in restricted regions, it may be possible to choose a mesh-system which follows more or less closely the lines of absolute structure in the world, and so simplify the phenomena which are related to it. But the world-structure is not of a kind which can be traced in an exact way by mesh-systems, and in any large region the mesh-system drawn must be considered arbitrary. In any case the systems used in current physics are arbitrary.[1] Arthur Stanley Eddington, 1920

In the spring of 1916, Albert Einstein published a synthetic account of the theory he had recently completed after eight years of intense efforts. This account, entitled *Die Grundlage der allgemeinen Relativitätstheorie*, is rightly regarded as a historical landmark since it served as the foundation of subsequent research on general relativity. Yet some features of Einstein's exposition made it ill-fitted for this service. Here is a short list: lack of a clear distinction between heuristic and deductive arguments, conceptual obscurities or contradictions, gaps and errors in some deductions, and an opaque non-geometrical approach to tensor calculus. Although these textual flaws did not necessarily reflect misunderstandings on Einstein's part and although they did not prevent him from obtaining essentially correct results, they confused his least receptive readers and they challenged the most perspicacious ones.[2]

Broad conceptual or philosophical aspects of this early reception of Einstein's *Grundlage* have been abundantly studied by historians and philosophers of physics. These aspects include general covariance and its relation to the equivalence principle (Howard, 1999; Norton, 1992, 1993; Stachel, 1993), Einstein's exploitation of Mach's principle (Barbour & Pfister, 1995; Janssen, 2004), the status of measuring rods and clocks in general relativity (Brown, 2005; Giovanelli, 2014; Ryckman, 2005), the expression of energy-momentum conservation (Brading, 2005; Cattani & De Maria, 1993), and the nature of the Schwarzschild singularity (Eisenstaedt, 1982). The present article is devoted to a more practical though less studied aspect of this reception: troubles with the meaning of coordinates—or Eddington's "mesh-system"—in the derivations of the three main predictions of early general relativity. This issue is not unrelated to the broader conceptual issues, and it is also linked to the well-studied history of the early experimental tests of general relativity.[3] This is why it has partially but penetratingly been addressed in Jean Eisenstaedt's history of the Schwarzschild singularity and in John Earman and Clark Glymour's

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[1] Eddington (1920b, p. 150).

[2] Einstein (1916).

[3] On the history of tests of general relativity, see Crelinsten (2006); Earman & Glymour (1980a, 1980b); Hentschel (1994, 1998); Kennefick (2007, 2009); Stanley (2003).

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history of tests of the gravitational redshift.[4] It nonetheless deserves to be considered separately and fully, not only because of the richness of the relevant historical materials but also because it touches basic difficulties in understanding the foundations of general relativity.

In retrospect, these difficulties have much to do with a much simpler problem: the characterization of the intrinsic geometry of an ordinary curved surface. This is why the first section of this article is a concise explanation of the meaning of coordinates and metric coefficients in the Gaussian theory of surfaces. The second section is a point-by-point discussion of Einstein's *Grundlage* of 1916, with emphasis on the following conceptual difficulties: conflation between reference frame and coordinate system, confusion between extended and local frames, implicit metric meaning of coordinates in some reasoning. These difficulties affected the comprehension of the three basic new predictions given in the three last of the fifty-five pages of Einstein's memoir. These predictions and early commentary or developments by other theorists are discussed separately in the three last sections of the present essay.

The first prediction is the redshift of the spectral lines from stars. Einstein's derivation is problematic, because it relies on the coordinate-dependent notion that clocks slow down in an intense gravitational field, a notion at odd with the rest of his theory. Hermann Weyl, Arthur Eddington, and Max von Laue explained that the gravitational redshift depended on the conservation of frequency (when measured with respect to the time coordinate for which the metric coefficients are time-independent) during the propagation of light from the star to the earth. Eddington and Laue proved this conservation. They also understood that the equivalence principle justified the assumption that an atomic oscillation corresponded to a well-defined value of the proper time d*s*. In addition, Laue gave proofs that spring-clocks and Bohr-atom clocks measured the proper time. None of these important remarks and results could be found in Einstein's *Grundlage.* They required a clarification of the meaning of the time coordinate in the static solutions of Einstein's field equations, as well as an empirical grounding of the proper time.

Einstein's derivation of the gravitational deflection of light was equally problematic. It tacitly assumed a partial persistence of the old metric interpretation of the coordinates; it was based on the unproven validity of Huygens's principle for propagation referred to a specific system of coordinates; and it contained an error in the expression of the local curvature of light rays (this error, which seems to have remained unnoticed, does not affect the final result). Weyl and Tullio Levi-Civita proved the validity of Fermat's equivalent principle for null geodesics, and Laue further proved that light followed null geodesics in the eikonal approximation. After a contribution by Ludwig Flamm, the usual route to the gravitational deflection of light has been through the limiting case of geodetic motion for which the velocity of the particle reaches the velocity of light. This procedure, which Eddington justified through the equivalence principle, avoids the difficulties of Einstein's approach except one: it remains to be shown that usual astronomic measurements yield the deflection expressed in the special system of coordinates of the Schwarzschild solution. Weyl and Eddington provided the missing argument.

Einstein's derivation of the anomaly in the advance of mercury's perihelion similarly lacks any discussion of the relation between the favored coordinates and astronomic observations. Here as in the light-deflection case, Einstein must have intuitively understood that the favored coordinates departed from the usual Newtonian coordinates in such a way that the measured parameters retained their usual meaning. Guido Beck clarified this point in his encyclopedia article of 1929. Several years earlier, in 1921, Paul Painlevé and Allvar Gullstrand had fallen into the error of confusing the coordinates of any given central-symmetric solution of Einstein's field equations with the astronomically measured coordinates, even when the former coordinates significantly differed from those used by Einstein and Schwarzschild. This was perhaps the most glaring manifestation of the difficulty that Einstein's readers (and the pre-1915 Einstein) had in assimilating the basic fact that the coordinates in general relativity have no *a priori* physical meaning independent of the expression of the metric coefficients.

As was mentioned, most of the difficulties encountered in the interpretation of the role of coordinates in general relativity have counterparts in the theory of ordinary curved surface. It is therefore no wonder that the two men who most significantly contributed to the clarification of Einstein's theory, Weyl and Eddington, also were the promoters of a geometrical understanding of the theory. When Einstein and his mathematician-friend Marcel Grossmann developed the mathematical apparatus of general relativity, they relied on the algebraic tradition of the "absolute differential calculus" of Elwin Bruno Christoffel and Gregorio Ricci Curbastro. This calculus was regarded as a general theory of quadratic differential forms and associated covariant quantities, a theory of which geometry was only one interesting application among many others.[5]

Levi-Civita and Weyl were among the first mathematicians to react to Einstein's new theory. In 1917, Levi-Civita re-injected geometric intuition into the theory with his concept of parallel transport. Weyl soon recast the theory in his *Nahegeometrie*, based on the concept of connection between local group structures. His *Raum · Zeit · Geometrie* of 1918 was both a treatise on the foundations of infinitesimal geometry and a systematic exposition of general relativity worthy of Einstein's admiration: "I am reading the proofs of your book…with true enthusiasm. It is like a master's symphony. Every little word relates to the whole, and the layout of the work is grandiose." Weyl's philosophical turn of mind helped him clarify the basic interpretive issues of the theory. In England, Eddington shared this quality and some of the mathematical brilliance. In addition, his competence as an astronomer helped him clarify the relation of the theory to concrete observation. As his well-known, in 1919 he organized the British solar-eclipse expeditions that were commonly believed to confirm Einstein's prediction of the gravitational deflection of starlight when passing close to the sun. His endeavor to explain general relativity to a broad audience brought him to dissolve the difficulties of the concept of metric manifold into simple geometric illustrations. His beautifully written books on general relativity, the more philosophical *Space, time and gravitation* of 1920 and the more technical *The mathematical theory of relativity* of 1923 long were the best English sources for learning general relativity, and they remain highly recommended readings to this day. Other important sources of interpretive lucidity were Max von Laue's articles and books on relativity theory, which combined technical prowess and philosophical profundity; the young Wolfgang Pauli's encyclopedia article of 1921, which concisely and competently synthesized anterior contributions to the theory; and Guido Beck's later encyclopedia article (1929), which gave special attention to the meaning of coordinates in observational predictions.[6]

[4] Eisenstaedt (1982) includes an instructive account of controversies on the meaning of coordinates in Einstein's derivation of the perihelion shift; Earman & Glymour (1980a) contains a detailed study of confusions in the early theoretical derivations of the gravitational redshift.

[5] Cf. Reich (1994). On Grossmann's contribution, cf. Sauer (2013).

[6] Levi-Civita (1917a); Weyl (1918); Einstein to Weyl, 8 Mar 1918, *ECP*, vol. 8; Beck (1929); Eddington (1920, 1923); Laue (1921); Pauli (1921). Weyl's and Eddington's extensions of Einstein's general relativity are here irrelevant. There