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# An argument for ψ-ontology in terms of protective measurements

Shan Gao

*Institute for the History of Natural Sciences, Chinese Academy of Sciences, Beijing 100190, China*

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ABSTRACT

The ontological model framework provides a rigorous approach to address the question of whether the quantum state is ontic or epistemic. When considering only conventional projective measurements, auxiliary assumptions are always needed to prove the reality of the quantum state in the framework. For example, the Pusey–Barrett–Rudolph theorem is based on an additional preparation independence assumption. In this paper, we give a new proof of ψ-ontology in terms of protective measurements in the ontological model framework. The proof does not rely on auxiliary assumptions, and it also applies to deterministic theories such as the de Broglie–Bohm theory. In addition, we give a simpler argument for ψ-ontology beyond the framework, which is based on protective measurements and a weaker criterion of reality. The argument may be also appealing for those people who favor an anti-realist view of quantum mechanics.



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## 1. Introduction

The nature of the quantum state has been a hot topic of debate since the early days of quantum mechanics. A long-standing question is whether the quantum state assigned to a single system represents the physical state of the system or a state of (incomplete) knowledge about the physical state of the system (Einstein, Podolsky, & Rosen, 1935). In recent years, the framework of ontological models provides a rigorous approach to address this question by formalizing the distinction between these two views, which are referred to as ψ-ontic and ψ-epistemic, respectively (Harrigan & Spekkens, 2010; Spekkens, 2005). Several theorems have also been proved to establish the ψ-ontic view within the framework (Colbeck & Renner, 2012; Hardy, 2013; Patra, Pironio, & Massar, 2013; Pusey, Barrett, & Rudolph, 2012).[1] However, on the one hand, the ontological model framework is not very general, and on the other hand, auxiliary assumptions are always required to prove these ψ-ontology theorems, e.g. the preparation independence assumption for the Pusey–Barrett–Rudolph theorem. It thus seems to be impossible to completely rule out ψ-epistemic models for quantum mechanics without auxiliary assumptions. Indeed, by removing the assumptions of these ψ-ontology theorems, explicit ψ-epistemic models can be constructed to reproduce the statistics of quantum measurements in Hilbert spaces of any dimension (Aaronson, Bouland, Chua, & Lowther, 2013; Lewis, Jennings, Barrett, & Rudolph, 2012).

In this paper, we will give a new argument for ψ-ontology in terms of protective measurements, first in the ontological model framework and then beyond the framework. Protective measurements are distinct from projective measurements in that a protective measurement can directly obtain the expectation value of the measured observable in the measured state with certainty (Aharonov, Anandan, & Vaidman, 1993; Aharonov & Vaidman, 1993), while a projective measurement can only obtain one of the eigenvalues of the measured observable with certain probability in accordance with the Born rule. As a consequence, the existence of protective measurements will extend the ontological model framework, and provide more resources for proving the reality of the quantum state.

The plan of this paper is as follows. In Section 2, we give a concise introduction to protective measurements. It is shown that the appearance of expectation value as a measurement result is quite natural when the measured state is not changed during the measurement as for protective measurements. In Section 3, we present a new, rigorous proof of ψ-ontology in the extended ontological model framework which includes protective measurements. The proof does not rely on auxiliary assumptions, and it

*E-mail address:* gaoshan@ihns.ac.cn

[1] For a comprehensive review of these ψ-ontology theorems and related work see Leifer (2014).

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also applies to deterministic theories such as the de Broglie–Bohm theory. In Section 4, we further improve the ontological model framework by replacing one of its fundamental assumptions with a more reasonable assumption. We argue that although the proofs of existing $\psi$-ontology theorems cannot go through under this new assumption, our proof is still valid. In Section 5, we replace the ontological model framework with a weaker criterion of reality, which is arguably an improved version of the Einstein–Podolsky–Rosen criterion of reality, and give a simpler argument for
$\psi$-ontology based on protective measurements and this criterion of reality. Conclusions are given in the last section.

## 2. Protective measurements

The existing $\psi$-ontology theorems and $\psi$-epistemic models are both based on an analysis of conventional projective measurements.[2] However, there are in fact other types of quantum measurements, one of which is the important but seemingly less-known protective measurements (Aharonov & Vaidman, 1993; Aharonov et al., 1993). During a protective measurement, the measured state is protected by an appropriate mechanism such as via the quantum Zeno effect, so that it neither changes nor becomes entangled with the state of the measuring device. In this way, such protective measurements can measure the expectation values of observables on a single quantum system, even if the system is initially not in an eigenstate of the measured observable, and the quantum state of the system can also be measured as expectation values of a sufficient number of observables.

By a projective measurement on a single quantum system, one obtains one of the eigenvalues of the measured observable, and the expectation value of the observable can only be obtained as the statistical average of eigenvalues for an ensemble of identically prepared systems. Thus it seems surprising that a protective measurement can obtain the expectation value of the measured observable directly from a single quantum system. In fact, the appearance of expectation value as a measurement result is quite natural when the measured state is not changed during the measurement as for protective measurements (Aharonov et al., 1993). In this case, the evolution of the combining state is

$$|\psi(0)\rangle|\phi(0)\rangle \to |\psi(t)\rangle|\phi(t)\rangle, \quad t>0 \tag{1}$$

where $|\psi\rangle$ denotes the state of the measured system and $|\phi\rangle$ the state of the measuring device, and $|\psi(t)\rangle$ is the same as $|\psi(0)\rangle$ up to a phase factor during the measurement interval $[0,\tau]$. The interaction Hamiltonian is given by $H_I = g(t)PA$, where $A$ is the measured observable, $P$ is the conjugate momentum of the pointer variable $X$ of the device, and the time-dependent coupling strength $g(t)$ is a smooth function normalized to $\int dt\, g(t) = 1$ during the measurement interval $\tau$, and $g(0) = g(\tau) = 0$. Then by Ehrenfest's theorem we have

$$\frac{d}{dt}\langle\psi(t)\phi(t)|X|\psi(t)\phi(t)\rangle = g(t)\langle\psi(0)|A|\psi(0)\rangle, \tag{2}$$

which further leads to

$$\langle\phi(\tau)|X|\phi(\tau)\rangle - \langle\phi(0)|X|\phi(0)\rangle = \langle\psi(0)|A|\psi(0)\rangle. \tag{3}$$

This means that the shift of the center of the pointer of the device gives the expectation value of the measured observable in the measured state. This analysis also shows that a protective measurement obtaining an expectation value is independent of the protection procedure.

That the quantum state of a single system can be measured by protective measurements can be illustrated with a specific example (Aharonov & Vaidman, 1993). Consider a quantum system in a discrete nondegenerate energy eigenstate $\psi(x)$. In this case, the measured system itself supplies the protection of the state due to energy conservation and no artificial protection is needed. We take the measured observable $A_n$ to be (normalized) projection operators on small spatial regions $V_n$ having volume $v_n$:

$$A_n = \begin{cases} \dfrac{1}{v_n} & \text{if } x \in V_n, \\ 0 & \text{if } x \notin V_n. \end{cases} \tag{4}$$

An adiabatic measurement of $A_n$ then yields

$$\langle A_n\rangle = \frac{1}{v_n}\int_{V_n} |\psi(x)|^2\, dv, \tag{5}$$

which is the average of the density $\rho(x) = |\psi(x)|^2$ over the small region $V_n$. Similarly, we can adiabatically measure another observable $B_n = (\hbar/2mi)(A_n\nabla + \nabla A_n)$. The measurement yields

$$\langle B_n\rangle = \frac{1}{v_n}\int_{V_n} \frac{\hbar}{2\mathrm{mi}}(\psi^*\nabla\psi - \psi\nabla\psi^*)\mathrm{d}v = \frac{1}{v_n}\int_{V_n} j(x)\,\mathrm{d}v. \tag{6}$$

This is the average value of the flux density $j(x)$ in the region $V_n$. Then when $v_n \to 0$ and after performing measurements in sufficiently many regions $V_n$ we can measure $\rho(x)$ and $j(x)$ everywhere in space. Since the quantum state $\psi(x,t)$ can be uniquely expressed by $\rho(x,t)$ and $j(x,t)$ (except for an overall phase factor), the above protective measurements can obtain the quantum state of the measured system.

## 3. My argument

Since the quantum state can be measured from a single system by a series of protective measurements, it seems natural to assume that the quantum state refers directly to the physical state of the system. Several authors, including the discoverers of protective measurements, have given similar arguments supporting this implication of protective measurements for the ontological status of the quantum state (Aharonov & Vaidman, 1993; Aharonov et al., 1993; Anandan, 1993; Dickson, 1995; Gao, 2013, 2014a; Hetzroni & Rohrlich, 2014). However, these analyses are not very rigorous and also subject to some objections (Dass & Qureshi, 1999; Schlosshauer & Claringbold, 2014; Unruh, 1994).[3] It is still debatable whether protective measurements imply the reality of the quantum state. In the following, we will give a new, rigorous argument for $\psi$-ontology in terms of protective measurements in the ontological model framework.

The ontological model framework is based on two fundamental assumptions (Harrigan & Spekkens, 2010; Pusey et al., 2012; Spekkens, 2005). The first assumption is that if a quantum system is prepared such that quantum mechanics assigns a pure state to it, then after preparation the system has a well-defined set of physical properties, which is usually represented by a mathematical object, $\lambda$. This assumption is necessary for the analysis of the ontological status of the quantum state, since if such physical properties don't exist, it will be meaningless to ask whether or not the quantum state describes them. The second assumption is that when a measurement is performed, the behavior of the measuring device is only determined by the complete physical state of the system, along with the physical properties of the measuring

[2] It is worth noting that the existing $\psi$-epistemic models reproduce only the statistics of conventional projective measurements, not yet the outcomes of protective measurements (Aaronson et al., 2013; Lewis et al., 2012; Spekkens, 2007).

[3] See Gao (2014b; 2016) for a review of these objections and also answers to them.