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## Becker–Blaschke problem of space<sup>☆</sup>

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### ABSTRACT

In a letter to Weyl, Becker proposed a new way to solve the problem of space in the relativistic context. This is the result of Becker's encounter with the two traditions of thinking about space: Husserlian transcendental phenomenology and Blaschke's equiaffine differential geometry. I reconstruct the mathematical content of the Becker–Blaschke solution to the problem of space and highlight the philosophical ideas that guide this construction. This permits me to underline some common properties of Riemannian and Minkowskian manifolds in terms of an unusual notion of isotropy. Finally, I will use this construction as a support to analyze several philosophical differences between Weyl's and Becker's proposals.

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## 1. Introduction

In his letter to Hermann Weyl on 10.10.1924, Oskar Becker sketched a new way of solving the relativistic problem of space. This solution is more adequate for his philosophical perspective than the solution presented in his habilitation thesis. It uses the techniques of Wilhelm Blaschke's equiaffine differential geometry. I have mathematically completed this construction, which I propose to call *the Becker–Blaschke problem of space*.

The first purpose of the current article is to present the mathematical reconstruction of this peculiar problem of space. By “reconstruction”, I mean that it was necessary to complete (and sometimes correct on some minor points) the arguments and proofs, which were very sketchy and fragmentary in the letter. Indeed, thanks to Weyl's mathematical culture and his knowledge of Husserl's and Becker's works, he could read between the lines. Moreover, as shown below, Becker makes some minor mistakes in the letter and some not perfectly justified claims. This is why, in

their editorial work, Mancosu and Ryckman did not reconstruct the Becker–Blaschke problem of space because “it is questionable whether his [Becker's] sketch is developed or coherent enough to merit discussion<sup>1</sup>”. I aim to show, however, that Becker's sketch can be completed and corrected in order to become a coherent mathematical construction. This reconstruction will provide new insights on the foundations of Minkowskian manifolds concerning the role of isotropy and the justification of the signature.

My second purpose is to use this construction to analyze some philosophical differences between Weyl and Becker. Among these differences, I will focus on the question of isotropy and on the question of the articulation between infinitesimal and finite metrical relationships. With regard to the role of the phenomenology of perception in the justification of the Pythagorean nature of the metric, I will provide my opinion about the difference between Becker and Weyl. A more complete justification of the last point, which would need a long development of Weyl's texts, will be postponed to a second article.<sup>2</sup>

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<sup>1</sup> See Mancosu & Ryckman (2005, pp. 170–171).

<sup>2</sup> See Bernard (2015b).

In the following sections of the article, I will first present the philosophical context of the debate. Then, I will discuss the general philosophical framework of Becker's investigations on the problem of space, and I will justify his need for a new solution to this problem. Thereafter, my reconstruction of the Becker–Blaschke problem of space and its solution will follow. Finally, I will use my reconstruction to check and clarify my claims about the philosophical oppositions between Becker and Weyl.

## 2. The philosophical context of the debate

### 2.1. Relativity theories and the idealistic thinkers

As is well known,<sup>3</sup> the emergence of the two theories of relativity forced the idealistic position on space to be renewed. Within the wide family of idealistic thinkers interested in relativity theory—academic philosophers or scientists, we found in the early 1920s the neo-Kantians,<sup>4</sup> Eddington, the transcendental phenomenologists (notably Husserl and Becker), and Weyl. They had a common enemy: empiricism. In opposition to it, they tried to highlight the importance of the transcendental use of some *a priori* elements of knowledge originating from the constitution of subjectivity in order to give foundations to mathematics and natural science. Within this wide common perspective, they disagree on the precise status one has to give to those *a priori* elements and to the involved notion of subjectivity.

Concerning the foundations of geometry and its physical application, the idealistic thinkers acknowledged with Kant the necessity of admitting the existence of an aprioristic notion of space. However, some of them were questioning the roles of intuition and sensibility in the determination of this aprioristic notion of space. Moreover, because of the historical evolution of geometry, all of the idealistic thinkers of this period had to insist, even more than Kant, on the epistemological difference between the aprioristic space and the space of mathematics and physics. The former is thought of as a condition of possibility of the latter, but they must not be confused.

There is no doubt that, already in the Kantian orthodoxy, the space of geometry in so far as it is structured by conceptual constructions (points, lines, metrics, etc.) is not identified with space as an *a priori* form of intuition. The space of geometry is the result of an activity of the spontaneous faculty of our understanding on pure intuitions given by our sensibility by the means of imagination. However, at least in some passages of his texts, Kant seemed to believe that there was a unique epistemic pathway that led us from space as pure form of our intuition to the supposedly unique conceptualized space of geometry, that of Euclid.<sup>5</sup> Even if this point has been often contested, it was the common view at the moment in German education, and Weyl subscribed to it.<sup>6</sup> Since the reflections of geometers of the XIXth century on non-Euclidean geometries and since their crucial application to

<sup>3</sup> See Ryckman (2005), Bitbol, Kerszberg, & Petitot (2010), and Hentschel (1990).

<sup>4</sup> Among the neo-Kantians of Marburg, Natorp (1910), Cassirer (1910), and Hönigswald (1909) wrote important books inspired by the special theory of relativity. Natorp retired in 1922 and died two years later. In the first years following the emergence of general theory, Cassirer (1921) (who was in Hamburg since 1919) and Cohn (1917) (in Freiburg) published important books on the subject. See Hentschel (1990, p. 199–ff) for more historical references.

<sup>5</sup> Kant (1998), in B64–66, claims that the propositions of geometry are synthetic *a priori* and that the only way for us to explain the possibility of such propositions is to posit that space is merely a subjective condition of all our outer intuitions. The name of Euclid does not appear, but Kant does not specify that he uses the term “geometry” with another meaning as the usual at that time, namely, Euclidean geometry.

<sup>6</sup> See for example Weyl (1954).

relativity theories, one had to give up the idea that there was a unique way to attach conceptual structures to space and then to articulate them with physical data. It was the end of the hegemony of Euclidean geometry as giving the truth on space structures. Therefore, the neo-Kantians and the other idealistic thinkers including Husserlian phenomenologists could only defend their position by criticizing some aspects of the Transcendental Aesthetic. Therefore, if some *a priori* elements were still necessary to understand the constitution of physical spatial relationships, there was not a unique way to express them within the theoretical construction. This was a common observation of both the idealistic thinkers and the thinkers that were becoming more and more favorable to empiricism, like Carnap, Reichenbach, and Schlick.<sup>7</sup>

We know, from direct references, that Cassirer was directly influenced by Weyl in his interpretation of relativity theories.<sup>8</sup> In return, Weyl and Becker both refer to Cassirer in their bibliographical notes.<sup>9</sup> The three authors wanted to show the need for a dialogue between philosophy and the sciences. More precisely, they wanted to reconcile the transcendental-idealistic position in philosophy<sup>10</sup> with a deep comprehension of the dynamic of physical truth throughout the evolution of theories; i.e., they wanted to show how idealistic *a priori* elements of knowledge participated in the construction of science, even if they were applied again and again in new forms because of the constructive (rather than purely intuitive) nature of theoretical knowledge.<sup>11</sup> This is the case, for Cassirer, for this particular *a priori* element that is *pure space*. He even avoids calling it a “form of intuition”, instead considering it as methodic presupposition prescribing rules of constitution of physical objects. In spite of the fundamental role of intuition for phenomenology, Becker and Husserl also avoided speaking of space as a “form of our intuition”, this expression hiding the fact that space is fundamentally a form of thinghood < Form der Dinglichkeit >.<sup>12</sup> In contrast, Weyl rehabilitates the Kantian expression.<sup>13</sup> Nevertheless, this does not necessarily mean that, according to Weyl, the metrical structure of physical space would be based on the psychophysical notion of perception or on its phenomenological correlate.

Cassirer (1921)<sup>14</sup> tries to identify, within the construction of general relativity, where the “methodic presupposition” that Kant called “pure intuition” precisely is. He found it in the general presupposition of space–time as a four-dimensional manifold independent of the particular system of coordinates we use to describe it. Therefore, pure space and time seem to be expressed, within the theory, by a purely topologico-differential schema. This is close to Weyl's position in the first edition of

<sup>7</sup> See, however, Ryckman (2005) for more details on the difference in status of aprioristic elements by those authors by comparison with the neo-Kantians and Weyl.

<sup>8</sup> See Cassirer (1921, pp. 397; 437; 454) in Cassirer (2004).

<sup>9</sup> See Weyl (2010, Chapter IV, bibliog. note. 1) and Becker (1923, p. 152, footnote 2).

<sup>10</sup> This is at least evident for Weyl in the early 1920s, even if he did not use the term “transcendental”, speaking about his own philosophy.

<sup>11</sup> Cassirer (1921) wrote:

[The fact that relativity theories compel us to give up some old presentational pictures of space and time] can affect the “pure intuition” of Kant only in so far as it is misunderstood as a mere picture and not conceived and estimated as a constructive method. (p.417)

Concerning Weyl's insistence on the constructive aspect of the foundations of scientific knowledge, see Weyl (1923a, pp. 45–46) and Sieroka (2009).

<sup>12</sup> See Husserl (1991, p. 43).

<sup>13</sup> Weyl (1919) speaks about space as a “form of our intuition” < Form unserer Anschauung >. In the same book, on p.10; 86, in Weyl (1918d,p. 385) and in Weyl (1923a, pp. 24; 43; 44) he speaks of space as a “form of appearances” < Form der Erscheinungen >.

<sup>14</sup> See on p. 417.

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