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Symmetries and the philosophy of language

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ABSTRACT

In this paper, I consider the role of exact symmetries in theories of physics, working throughout with the example of gravitation set in Newtonian spacetime. First, I spend some time setting up a means of thinking about symmetries in this context; second, I consider arguments from the seeming undetectability of absolute velocities to an anti-realism about velocities; and finally, I claim that the structure of the theory licences (and perhaps requires) us to interpret models which differ only with regards to the absolute velocities of objects as depicting the same physical state of affairs. In defending this last claim, I consider how ideas and resources from the philosophy of language may usefully be brought to bear on this topic.

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1. Introduction

The question of how the symmetries of a theory bear upon its representational content has been a subject of much recent discussion.[1] It is the contention of this paper that models related by a symmetry transformation are merely different ways of representing the same physical state of affairs (at least, with respect to qualitative properties); and that utilising resources from the philosophy of language provides an insightful way of defending this claim. This is both because it can provide us with new arguments to motivate such an interpretational stance, and because it can illuminate what structural features of such pairs of models make this interpretational stance permissible.

The structure of the paper is as follows. In Section 2, I outline the theory that will be our worked example throughout the paper: namely, that of Newtonian gravity set in (full) Newtonian spacetime,[2] or "Newtonian gravitation" (NG) for short. In Section 3, I outline some apparatus for approaching the symmetries of this theory (an apparatus which should generalise to other similar theories); and in Section 4, I discuss how models related by different kinds of symmetry relate to one another. With this much setting-up done, I turn in Section 5 to consider why models related by boosts should be taken to represent observationally identical states of affairs, and why this licences the dismissal of absolute velocities from our ontology. In Section 6, I go on to argue—against the received wisdom—that we can implement this dismissal without altering our theory, i.e., merely by making acceptable interpretational stipulations regarding the theory. In Section 7, I discuss the situations in which such an interpretational strategy would be advantageous. Finally, in Section 8, I consider whether such an interpretational stance may in fact be not merely acceptable, but positively required—at least if an unpleasant indeterminacy of reference is to be avoided. It is in Sections 6 and 8, in particular, that we will see how ideas from the philosophy of language (specifically, ideas regarding synonymy and translation) may be usefully borrowed for the purposes of philosophy of physics.

Two final remarks before we begin. I will approach NG via its models: that is, by specifying what kinds of mathematical structures will count as *kinematically possible models*, and then picking out a subset of those as *dynamically possible models*. The kinematically possible models are, roughly speaking, objects of the right mathematical type to represent a physically possible world; one can think of them as representing the metaphysically possible worlds. The dynamically possible models are then those which do, in fact, represent physically possible worlds. In this paper, I will assume that all dynamically possible models represent physically possible worlds. (This is not a truism: some views impose

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[1] See, for example, Saunders (2003b), Dasgupta (2009), or Belot (2013).

[2] That is, Newtonian rather than neo-Newtonian (aka Galilean) spacetime.

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metaphysical constraints which mean that some dynamically possible models represent metaphysically impossible, and *a fortiori* physically impossible, worlds.)[3]

2. Newtonian gravitation

Consider the theory of Newtonian gravitation, set in (full) Newtonian spacetime. The kinematically possible models of this theory are of the form $\langle \mathcal{A}, \mathcal{T}, \phi, \{\langle \mathbf{x}_i, m_i\rangle\}_{i\in I}\rangle$, where

- $\mathcal{A}$ is a three-dimensional (Euclidean) metric affine space (that is, a set of points X equipped with a faithful, transitive action of some (Euclidean) normed vector space V on X);
- $\mathcal{T}$ is a simply connected, one-dimensional manifold equipped with a metric and orientation;
- ϕ is a scalar field on $\mathcal{A}\times\mathcal{T}$; and
- $\{\langle \mathbf{x}_i, m_i\rangle\}_{i\in I}$ is a set of ordered pairs, each consisting of a smooth function $\mathbf{x}_i : \mathcal{T}\to\mathcal{A}$ and a scalar $m_i \in \mathbb{R}$ (with I just being an index set)

In such a model, $\mathcal{A}$ represents absolute space and $\mathcal{T}$ represents absolute time. I will refer to the product space $\mathcal{A}\times\mathcal{T}$ as the (Newtonian) *spacetime structure*, and abbreviate it as N. The terminology here is a little unhappy: the structure presented is referred to by Penrose[4] as *Aristotelian spacetime*, which—as we shall come to later—is used in the philosophy of physics literature to mean something else. So instead, I follow Saunders[5] in calling it Newtonian spacetime. This also accords with the influential terminology of Friedman[6] and Earman[7]—at least, insofar as what they and I call "Newtonian spacetime" match in their structural essentials. However, there are some differences in the manner of construction, which raise some issues: these are discussed further in Section 7.

ϕ and $\{\langle \mathbf{x}_i, m_i\rangle\}_{i\in I}$ represent, respectively, the gravitational potential and the gravitating particles (with the ith particle having mass m_i and trajectory $\mathbf{x}_i$); I will refer to these structures as the *dynamical structure*, and abbreviate them as P. Given that both $\mathcal{A}$ and $\mathcal{T}$ are equipped with metrics, it is straightforward to define the velocity $\dot{\mathbf{x}}_i$ and acceleration $\ddot{\mathbf{x}}_i$ of a particle, and the gradient $\nabla\phi$ and Laplacian $\nabla^2\phi$ of the potential. In order for a kinematically possible model $\mathcal{M} = \langle N, P\rangle$ to be a dynamically possible model, the dynamical structure P must satisfy the following equations for any $\mathbf{x}\in\mathcal{A}$ and $t\in\mathcal{T}$:

$$\ddot{\mathbf{x}}_i(t) = -\nabla\varphi(\mathbf{x}_i, t) \tag{1a}$$

$$\nabla^2\varphi(\mathbf{x}, t) = 4\pi G\sum_i m_i\delta(\mathbf{x}-\mathbf{x}_i) \tag{1b}$$

3. The symmetries of Newtonian gravitation

The symmetries of the theory sketched above are, of course, well-known; as is the fact that they come in two important classes. The *spacetime symmetries* of a given Newtonian spacetime N are the automorphisms of N. Since N is a product space $\mathcal{A}\times\mathcal{T}$, an automorphism f of N will map $(\mathbf{x}, t)\mapsto(f_{\mathcal{A}}(\mathbf{x}), f_{\mathcal{T}}(t))$, where $f_{\mathcal{A}}$ and $f_{\mathcal{T}}$ are automorphisms (i.e., isometries) of $\mathcal{A}$ and $\mathcal{T}$ respectively. Such isometries consist of translations, reflections and (in the case of $\mathcal{A}$) rotations.[8]

In this essay, I will be concerned with only the continuous symmetries.[9] Thus, the spacetime symmetries we are interested in consist of the translations (both temporal and spatial) and spatial rotations. The set of such symmetries for one Newtonian spacetime are referred to as the *Newton group* for that spacetime. The Newton group of any Newtonian spacetime is isomorphic to that for any other; consequently, we can think of them as faithful representations of a single abstract Newton group.

Towards introducing the second kind of symmetry, note that if we apply a member of the Newton group (or one of the discrete spacetime symmetries) to the *dynamical* structure of a model, we always obtain an isomorphic model. That is, given a kinematically possible model $\langle N, P\rangle$, define the image of $P = \langle\phi, \{\langle m_i, \mathbf{x}_i\rangle\}\rangle$ under a map $d : N\to N$ as $d[P] = \langle\phi', \{\langle m_i, \mathbf{x}_i'\rangle\}\rangle$, where

$$\phi'(\mathbf{x}, t) = \phi(d_{\mathcal{A}}^{-1}(\mathbf{x}), d_{\mathcal{T}}^{-1}(t)) \tag{2a}$$

$$\mathbf{x}_i'(t) = d_{\mathcal{A}}(\mathbf{x}_i(d_{\mathcal{T}}^{-1}(t))) \tag{2b}$$

If d is a member of the Newton group, then $\langle N, d[P]\rangle = \langle d[N], d[P]\rangle$, and so is isomorphic to $\langle N, P\rangle$. Because the conditions picking out dynamically possible models of NG are purely structural, they apply to any given model $\mathcal{M}$ if and only if they also apply to any model $\mathcal{M}'$ which is isomorphic to $\mathcal{M}$.

Thus, for any kinematically possible model $\langle N, P\rangle$ and any member d of the Newton group for N, $\langle N, P\rangle$ is dynamically possible if and only if $\langle N, d[P]\rangle$ is. Let us introduce a little terminology, and say that two models $\mathcal{M}$ and $\mathcal{M}'$ are *co-dynamical* if either both or neither are dynamically possible. We can then say that applying any member of the Newton group to a model yields a co-dynamical model. However, the Newton group is not the only set of continuous transformations with this kind of feature: the dynamical (im)possibility of any kinematically possible model is preserved under *boosts*.

A boost in N is specified by any vector v from the vector space V underlying $\mathcal{A}$ and any time $t_0\in\mathcal{T}$; the associated boost is then $b : (\mathbf{x}, t)\in N\mapsto(\mathbf{x}+\tau\mathbf{v}, t)\in N$, where τ is the oriented[10] distance between t_0 and t. If we add such boosts to the Newton group for N, we obtain what is known as the *Galilei group* for N; as before, we will also refer to an abstract Galilei group, of which the Galilei group for any given spacetime is a faithful representation. Boosts are *not* spacetime symmetries: a boost cannot be decomposed into a (single) automorphism of $\mathcal{A}$ and an automorphism of $\mathcal{T}$. Nevertheless, it is easy enough to check that $\langle N, P\rangle$ satisfies Eqs. (2) if and only if $\langle N, b[P]\rangle$ does so, for any boost b. As such, a boost—like a member of the Newton group—is a *dynamical symmetry*, a mapping whose application to (just) the dynamical structure of a kinematically possible model yields a co-dynamical model. It follows that each member of the Galilei group for N is a dynamical symmetry.

The Galilei group is not the full set of dynamical symmetries for NG: in addition to the discrete symmetries, one can apply time-dependent accelerations which are accompanied by appropriate alterations to the gravitational potential.[11] However, all the members of the Galilei group are dynamical symmetries, even though some of them (viz., the boosts) are not spacetime

[3] Such as Maudlin's metrical essentialism (Maudlin, 1988).

[4] Penrose (2004, chap. 17).

[5] Saunders (2013).

[6] Friedman (1983).

[7] Earman (1989).

[8] Strictly, $\mathcal{T}$ is also invariant under rotations; but because it is one-dimensional, the only "rotation" is the identity map.

[9] See Pooley (2003) and Huggett (2003) for discussion of some of the issues raised by spatial reflection symmetries (and by violations of those symmetries).

[10] That is, if t_0 precedes t with respect to the orientation of $\mathcal{T}$, then $\tau > 0$, and if t precedes t_0, then $\tau < 0$.

[11] For further discussion of this symmetry, see Saunders (2003a) or Knox (2014).