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Reduction and emergence in the fractional quantum Hall state

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ABSTRACT

We present the fractional quantum Hall (FQH) effect as a candidate emergent phenomenon. Unlike some other putative cases of condensed matter emergence (such as thermal phase transitions), the FQH effect is not based on symmetry breaking. Instead FQH states are part of a distinct class of ordered matter that is defined topologically. Topologically ordered states result from complex long-ranged correlations between their constituent parts, such that the system displays strongly irreducible, qualitatively novel properties.

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1. Introductory remarks

The *fractional quantum Hall effect* (FQHE) is a type of collective behaviour realized in a 2D system of electrons. At low temperatures an interacting ensemble of electrons realises a fluid-like state. In the presence of a magnetic field applied with a specific magnitude, the longitudinal resistivity becomes exponentially small and Hall resistance of the 2D fluid shows plateaux. Each plateau occurs at a value of resistivity determined by a given value of the so-called filling factor ν, which describes a ratio of filled to vacant electronic states. In cases when ν takes an integer number we observe *the integer quantum Hall effect* (IQHE); in which the Hall resistivity is found to be quantised in units of h/e^2. However, when ν is a ratio of integers (frequently one with an odd denominator) then we observe the FQHE, which involves the fluid-like electron liquid showing a form of generalised rigidity, along with an unusual spectrum of excitations involving fractional quantum numbers. That is to say, the excitations of a FQH system carry fractions of an electronic charge and are neither bosons nor fermions. Despite their similar names, the FQHE is characteristic of different phases of condensed matter to the IQHE, with this difference arising owing to the role of the interactions between electrons in the former case. Although most phases of condensed matter can be characterized by symmetry considerations, the FQH state is instead characterized by *topological order*. The discovery and theoretical elucidation of the FQHE won Robert Laughlin (1983), Horst Störmer and Daniel Tsui [Tsui, Störmer, and Gossard (1982)] the 1998 Nobel Prize in Physics.

The aim of this paper is twofold. Firstly, we introduce the rich physics of the FQHE to a philosophical audience. In order to facilitate this, we will go into the physics of the FQHE in some detail. We hope, given the unfamiliarity of topological states of matter to many philosophers of physics, that readers will be understanding if more technical details are included than would usually be the case when discussing a more familiar example (such as superconductivity). Secondly, within physics the FQHE is often considered a paradigmatic case of emergence. We attempt to connect the physicists' conception of emergence to philosophical notions of emergence. We will therefore aim to categorize the ways in which the FQHE can be said to be an emergent phenomenon. Our conclusion is that the presence of topological order in the FQHE is indicative of an intrinsic holism to the FQH system. Because of this, the FQHE bears serious consideration as an example of a metaphysically significant, "strongly" emergent phenomenon.

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2. Ways of characterising emergence

Condensed matter physics is one of the arenas in which there is a renewed interest in the notion of emergence. The often cited starting point for this resurgence of "New Emergentist" thinking is Anderson's (1972) paper *More is Different*. Anderson claims many condensed matter systems are characterized by novel particular systemic properties. Since then many authors have argued that physics provides examples of emergent phenomena (e.g. see Batterman, 2002; Bedau, 2008; Laughlin & Pines, 2000; Morrison, 2012).[1] The senses of emergence argued for by these authors suggest different degrees of metaphysical significance. For example: Bedau argues for weak emergence, which is a failure of predictability due to inherent complexity and contingency. Batterman is primarily concerned with explanatory emergence; where explanations fail to reduce to explanations expressed only in the vocabulary of a lower level description. Morrison is concerned with the ability of higher-level phenomena to bring about new properties of matter, such as spontaneous symmetry breaking within an electromagnetic gauge theory, resulting in superconductivity.

By contrast other authors (Howard, 2007; Humphreys, 1997; Silberstein & McGeever, 1999; Teller, 1986) have looked towards quantum phenomena such as entanglement as the best candidate for emergence, while Mark Wilson (1993) and Paul Mainwood (2006) have suggested that there are parallels between the emergentism of Anderson and the views expressed by the British Emergentists of the early 20th century such as Broad (1925). Inspired by these various accounts we will define four ways emergence can be spelled out; each depending on the way the relationship between parts and wholes is considered. This is far from an exhaustive list of the ways emergence can be thought of, but it will provide a simplified framework by which to compare claims about the FQHE.

Emergence 0: (E0) *Failure of inter-theoretic reduction or failure of explanatory reduction*. This is concerned with the properties appearing in two different descriptions of the same composite system. Are there features of system *S* that can only be explained by referring to *S* in terms of a level specific vocabulary, or can all theoretical predictions and explanations ultimately be spelt out in terms of microphysical theories/models/explanatory strategies?
Emergence 1: (E1) *Entity emergentism*. Entities are known to us through their indispensable role in prediction, explanation and manipulation (cf. Hacking, 1988). If a composite system produces novel entities which fulfil this role then these entities are ontologically robust.
Emergence 2: (E2) Novelty of systemic properties of composites compared with the components of those composites in isolation (or other, different, composites). This is the position taken by British emergentists such as Broad and by New Emergentists such as Anderson, Laughlin & Pines.
Emergence 3: (E3) Emergence as a failure of mereological supervenience. This position is concerned with properties of whole systems that are novel relative to the properties of the parts of that composite whilst part of that whole. One possibility is this novelty manifests itself as a new set of causal powers.

These different notions, whilst distinct, are not mutually exclusive, for example, belief in E2 may imply E0 (although E0 certainly does not imply E2). In this paper we leave aside E0 emergence. This is because: (1) we agree with Butterfield and Isham (1999) and Mainwood (2006) that the syntactic form of inter-theory reduction is too flexible a notion to be an interesting sense of emergence. (2) By contrast, although we think that a failure of *explanatory reduction* is an interesting sense of emergence, we want to focus on emergent features potentially unique to topological phases of matter. In short, although we believe FQHE is E0 emergent (in the explanatory sense), we believe that E0 emergence is widespread and discussions of it can be motivated by considering cases involving more familiar physics (e.g. thermal phase transitions). As such we will focus the discussion by comparing the FQHE to the senses of emergence captured by E1, E2, and E3.

3. The physics of the FQHE

In this section we will review the physics of the FQHE before moving on to philosophical discussion in Section 4. Since we suspect the particularities of the FQHE will be unfamiliar to many readers we will spend some time discussing detailed aspects of the physics. At the end of this Section 3.7 there will be a brief re-cap which will summarise the key points of the physics.

3.1. The three flavours of the Hall Effect

This paper is based on a simple experiment shown schematically in Fig. 1. When an electrical current passes through a conductor which lies in a transverse magnetic field, it is found that a voltage develops perpendicular to both the current and the magnetic field directions. This is known as the Hall effect (see Singleton, 2001). The effect has a simple explanation based on the physics of classical electromagnetism: the moving electrons constituting the current feel a Lorentz force acting perpendicular to both their velocity and the magnetic field direction. This force deflects the electrons and causes electric charge to build up on the walls of the conductor, as shown in Fig. 1. The build-up of charge creates an electric field E_y that opposes the Lorentz force, allowing other electrons to pass though the conductor. However, the result of this build-up of static charge is the transverse Hall voltage. Historically, the Hall effect was important in the development of our understanding of the physics of solid materials. However, in the last 40 years it has also been key to the development of our understanding of many-body quantum mechanics. This arose from the possibility to synthesise semiconductors in which electrons are constrained to move in two dimensions (2D) only. These semiconductor structures energetically confine electrons in a deep potential well in one spatial dimension, but do not constrain their motion in the other two. When the Hall effect experiment is repeated on such a system (using the geometry shown in Fig. 1(b)), with a magnetic field directed perpendicular to the 2D plane, we measure *quantised* behaviour in the Hall effect, indicative of well-resolved quantum mechanical energy levels. Specifically, we apply the magnetic field B in the z-direction and drive a current J_x in the x-direction, measuring the transverse (or Hall) resistivity $\rho_{xy} = E_y / J_x$ [Fig. 2(a) and (b)].

The most striking results is that ρ_{xy} exhibits plateaux as a function of applied magnetic field [Fig. 2(b)]. We also find that the longitudinal resistivity $\rho_{xx} = E_x / J_x$ becomes vanishingly (or immeasurably) small along the flat sections of the plateaux, taking non-zero values only in the regions between plateaux [Fig. 2(a)]. This observation is known as the integer quantum Hall effect (IQHE) (Singleton, 2001). The explanation for this physics is essentially a quantum mechanical (momentum-) space filling argument. The system comprises mobile electrons, which are fermions and hence are prevented by the exclusion principle from occupying the same states as other electrons. There are only a

[1] Of course there are many other areas where emergence is discussed, such as the philosophy of mind and chemistry (e.g. Gibb, 2012; Hendry, 2010; O'Connor & Wong, 2005).