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Interpreting the Modal Kochen–Specker theorem: Possibility and many worlds in quantum mechanics

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ABSTRACT

In this paper we attempt to physically interpret the Modal Kochen–Specker (MKS) theorem. In order to do so, we analyze the features of the possible properties of quantum systems arising from the elements in an orthomodular lattice and distinguish the use of "possibility" in the classical and quantum formalisms. Taking into account the modal and many worlds non-collapse interpretation of the projection postulate, we discuss how the MKS theorem rules the constraints to actualization, and thus, the relation between actual and possible realms.

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1. Introduction

In classical physics, every physical system may be described exclusively by means of its *actual properties*, taking 'actuality' as expressing the *preexistent* mode of being of the properties themselves, independently of observation—the 'pre' referring to its existence previous to measurement. The evolution of the system may be described by the change of its actual properties. Mathematically, the state is represented by a point (p, q) in the corresponding phase space Γ and, given the initial conditions, the equation of motion tells us how this point evolves in Γ.[2] Physical magnitudes are represented by real functions over Γ. These functions commute with each other and can be all interpreted as possessing definite values at any time, independently of physical observation. In this scheme, speaking about potential or possible properties usually refers to functions defined on points in Γ to which the state of the system will arrive at a future instant of time; these points, in turn are completely determined by the equations of motion and the initial conditions.

In the orthodox formulation of quantum mechanics (QM), the representation of the state of a system is given by a ray in Hilbert space $\mathcal{H}$. Contrary to the classical scheme, physical magnitudes are represented by operators on $\mathcal{H}$ that, in general, do not commute. This mathematical fact has extremely problematic interpretational consequences for it is then difficult to affirm that these quantum magnitudes are *simultaneously preexistent*. In order to restrict the discourse to sets of commuting magnitudes, different Complete Sets of Commuting Operators (CSCOs) have to be chosen. This choice has not found until today a clear justification and remains problematic. In the literature this feature is called *quantum contextuality*—it will be discussed in Section 2. Another fundamental feature of QM is due to the linearity of the Schrödinger equation which implies the existence of entangled states involving the measuring device. The path from such an entangled state, i.e. a superposition of eigenstates of the measured observable to the eigenstate corresponding to the measured eigenvalue is given, formally, by an axiom added to the formalism: *the projection postulate*. In Section 3 we will discuss the different physical interpretations of this postulate which is, either thought in terms of a "collapse" of the wave function (i.e., as a real physical interaction) or in terms of non-collapse proposals, such as the modal and many worlds interpretations. After having introduced and discussed these two main features of QM we will present, in

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[2] For simplicity, we have in mind a system that is only a material point.

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Section 4, our formal analysis regarding possibility in orthomodular structures. In Section 5, we shall discuss and analyze the distinction between mathematical formalism and physical interpretation, a distinction which can raise many pseudo-problems if not carefully taken into account. As a consequence of this distinction we will also put forward the difference between 'classical possibility' and 'quantum possibility'. In Section 6, we are ready to advance towards a physical interpretation of both quantum possibility and the MKS theorem—taking into account the specific formal constraints to modality implied by it. In Section 7 we will discuss the consequences of the MKS theorem regarding the many worlds interpretation. Finally, in Section 8, we provide the conclusions of our work.

2. Quantum contextuality and modality

The idea that a *preexistent* set of definite properties constitutes or describes reality is one of the basic ideas which remains the fundament of all classical physical theories and determines the possibility to speak about an independent objective world, a world which does not depend on our choices or consciousness. Physical reality can be then conceived and analyzed in terms of a theory—which describes a preexistent world—independently of actual observation. But, as it is well known, this description of physical reality faces several difficulties when presupposed in the interpretation of the quantum formalism. In formal terms, this is demonstrated by the Kochen–Specker (KS) theorem, which states that if we consider three physical magnitudes represented by operators **A**, **B** and **C**, with **A** commuting with **B** and **C** but **B** non-commuting with **C**, the value of **A** depends on the choice of the context of inquiry; i.e. whether **A** is considered together with **B** or together with **C** (Kochen & Specker, 1967). From an operational point of view, this is bypassed by considering the context (in KS sense) as the experimental arrangement—in line with the original idea of N. Bohr. However, if we attempt to go beyond the discourse regarding measurement results and provide some kind of realist representation of what is going on according to QM, we need to make sense of the indeterminateness of properties. As Chris Isham and Andreas Döring clearly point out:

> "When dealing with a closed system, what is needed is a realist interpretation of the theory, not one that is instrumentalist. The exact meaning of 'realist' is infinitely debatable but, when used by physicists, it typically means the following:
>
> 1. The idea of 'a property of the system' (i.e., 'the value of a physical quantity') is meaningful, and representable in the theory.
> 2. Propositions about the system are handled using Boolean logic. This requirement is compelling in so far as we humans think in a Boolean way.
> 3. There is a space of 'microstates' such that specifying a microstate leads to unequivocal truth values for all propositions about the system. The existence of such a state space is a natural way of ensuring that the first two requirements are satisfied.
>
> The standard interpretation of classical physics satisfies these requirements, and provides the paradigmatic example of a realist philosophy in science. On the other hand, the existence of such an interpretation in quantum theory is foiled by the famous Kochen–Specker theorem." (Döring & Isham, 2008, p. 2).

Contextuality can be directly related to the impossibility to represent a piece of the world as constituted by a set of definite valued properties independently of the choice of the context. This definition makes reference only to the actual realm. But as we know, QM makes probabilistic assertions about measurement results. Therefore, it seems natural to assume that QM does not only deal with actualities but also with possibilities. Then the question arises whether the space of possibilities is subject to the same restrictions as the space of actualities. Formally, on the one hand, the set of actualities is structured as the orthomodular lattice of subspaces of the Hilbert space of the states of the system and, as Michael Dickson remarks in Dickson (2001), the KS theorem (i.e., the absence of a family of compatible valuations from subalgebras of the orthomodular lattice to the Boolean algebra of two elements **2**) can be understood as a consequence of the failure of the distributive law in the lattice. On the other hand, given an adequate definition of the possibility operator $\Diamond$—as the one developed in bib14,bib16—the set of possibilities is the center of an enlarged structure. Since the elements of the center of a structure are those which commute with all other elements, one might think that the possible propositions defined in this way escape from the constraints arising from the non-commutative character of the algebra of operators. Thus, at first sight one might assume that possibilities behave in a classical manner.

When predicting measuring results the context has been already fixed. However, probability is a measure over the whole lattice and, consequently, the set of events over which the measure is defined is non-distributive, calling attention to the interpretation of possibility and probability. As noticed by Schrödinger in a letter to Einstein (Bub, 1997, p. 115): "It seems to me that the concept of probability is terribly mishandled these days. Probability surely has as its substance a statement as to whether something *is* or *is not* the case—of an uncertain statement, to be sure. But nevertheless it has meaning only if one is indeed convinced that the something in question quite definitely *is* or *is not* the case. A probabilistic assertion presupposes the full reality of its subject." Also von Neumann was worried about a sound definition of probability, as mentioned in Rédei (2001).[3] The difficulties with a rigorous definition of probability made von Neumann abandon the orthodox formalism of QM in Hilbert space to which he himself had so much contributed and face the classification of the factors and their dimension functions which led to the subject of von Neumann's algebras.[4]

In order to explicitly verify whether modal propositions escape from KS-type contradictions, in previous works we have developed a mathematical scheme which allowed us to deal with both actual and possible propositions in the same structure.[5] Within this frame

[3] As Rédei (2001, p. 157) states: "To see why von Neumann insisted on the modularity of quantum logic, one has to understand that he wanted quantum logic to be not only the propositional calculus of a quantum mechanical system but also wanted it to serve as the event structure in the sense of probability theory. In other words, what von Neumann aimed at was establishing the quantum analogue of the classical situation, where a Boolean algebra can be interpreted both as the Tarski–Lindenbaum algebra of a classical propositional logic and as the algebraic structure representing the random events of a classical probability theory, with probability being an additive normalized measure on the Boolean algebra."

[4] It might be argued that a complete theory of quantum probability is still lacking. On the one hand, type II_1 factor (the one whose projection lattice is a continuous geometry, and thus an orthomodular modular lattice as required by a definition of probability) is not an adequate structure to represent quantum events. On the other hand, there exist different candidates for defining conditional probability and there is not a unique criterium for choosing among them (Dalla Chiara, Giuntini, & Greechie, 2004). Moreover, there are situations in which the frequentist interpretation does not apply and consequently it is required to develop new probability structures to account for quantum phenomena (Döring & Isham, 2012).

[5] Van Fraassen distinguishes two different isomorphic structures for dealing with possible and actual properties (Van Fraassen, 1991, Chapter 9). The main aspects of van Fraassen's modal interpretation in terms of quantum logic are as follows. The probabilities are of events, each describable as 'an observable having a certain value', corresponding to value states. If w is a physical situation in which system X exists, then X has both a *dynamic state* φ and a *value state* λ, i.e. $w = \langle \varphi, \lambda \rangle$. A *value state* λ is a map of observable **A** into non-empty Borel sets σ such that it assigns $\{1\}$ to $1_\sigma \mathbf{A}$. 1_σ is the characteristic function of the set σ of values. So, if the observable $1_\sigma \mathbf{A}$ has value 1, then it is impossible that **A** has a value outside σ. The