Studies in History and Philosophy of Modern Physics 48 (2014) 109–123

Contents lists available at ScienceDirect

Studies in History and Philosophy of Modern Physics

journal homepage: www.elsevier.com/locate/shpsb

Against dogma: On superluminal propagation in classical electromagnetism

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ARTICLE INFO

Article history:
Received 7 June 2014
Accepted 22 August 2014
Available online 11 October 2014

Keywords:
Electromagnetism
Superluminal propagation
Special relativity
Gordon metric

ABSTRACT

It is deeply entrenched dogma that relativity theory prohibits superluminal propagation. It is also experimentally well-established that under some circumstances, classical electromagnetic fields propagate through a dielectric medium with superluminal group velocities and superluminal phase velocities. But it is usually claimed that these superluminal velocities do not violate the relativistic prohibition. Here I analyze electromagnetic fields in a dielectric medium within a framework for understanding superluminal propagation recently developed by Geroch (1996, 2011) and elaborated by Earman (2014). I will argue that for some parameter values, electromagnetic fields *do* propagate superluminally in the Geroch–Earman sense.

When citing this paper, please use the full journal title *Studies in History and Philosophy of Modern Physics*

1. Introduction

Few dogmas in modern physics are as well-entrenched as the one stating that relativity theory prohibits superluminal propagation. And yet, despite its crucial importance to many physical arguments—foundational and otherwise—it is not fully clear what the status of this would-be prohibition is within relativity theory. Is it physical fields, such as electromagnetic fields or Klein–Gordon fields, that cannot propagate superluminally? Or is it energy-momentum? Or is it some variety of superluminal *signaling* that is prohibited? If the latter, then is there some unambiguous physical criterion for what constitutes a signal, or does signaling depend essentially on the possible transmission of information—perhaps between intentional beings? And whatever the details, is the prohibition on superluminal propagation supposed to be a *consequence* of relativity? Or is it a brute assumption, independent of the rest of the theory?

These questions are not idle quibbling about definitions. The idea that relativity theory, in some sense or another, prohibits superluminal propagation directly influences physicists' theoretical understanding of physical processes and their interpretation of experimental results. The prohibition also plays a central role in attempts to reconcile quantum physics with relativity. Moreover, there are concrete cases where the ambiguity concerning precisely what it is that relativity is meant to prohibit has led to confusion in the physics literature. For instance, in the context of experiments concerning light pulses in dielectric media, which I will discuss in more detail below, various apparently superluminal effects have been observed.[1] In such cases, it is ubiquitous practice to provide some argument for why the observed superluminal phenomena do not constitute superluminal propagation of a sort that would conflict with relativity. But these arguments have a decidedly *ad hoc* flavor and relatively little attention is paid, at least in this literature, to the more principled questions of what *would* constitute superluminal propagation of the troubling sort and how, in these particular cases, relativity manages to forbid it. At the very least, although relativity is often mentioned, a satisfactory relativistic treatment of the systems in question is rarely, if ever, on offer.

This is not to say that the more principled question is never taken up. In recent work, Geroch (2011) and Earman (2014) have articulated a precise and general account of what it would mean for a physical system to propagate superluminally in relativity theory.[2] More strikingly, Geroch, at least, argues that such fields should be understood as compatible with relativity theory, and

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http://dx.doi.org/10.1016/j.shpsb.2014.08.005
1355-2198/

[1] Here I limit attention to cases where the electromagnetic field is treated classically. Examples of purportedly superluminal phenomena multiply if one considers quantum electrodynamics. See Butterfield (2007) for a discussion of these examples aimed at philosophers.

[2] Weinstein (2006) may be seen as a sympathetic precursor to the view recently defended by Geroch and Earman. The principal difference, if one exists at all, concerns the role of "causal cones" (see Section 4, below) in the criterion of (maximal) field propagation velocity.

both Earman and Geroch present examples of fields that are, in some sense, "relativistic," and yet which exhibit superluminal propagation according to their criterion.[3]

My goal in the present paper is simply to bring these two literatures together. Along the way, I will defend three theses. First, I will argue that the sense of superluminal propagation developed by Geroch and elaborated by Earman can be understood as making precise a notion of propagation already present in the literature on electromagnetic fields in a dielectric—namely, superluminal values of the so-called Sommerfeld–Brillouin "wavefront velocity" associated with a field.[4] Second, I will argue that in at least one highly idealized case, on a fully relativistic treatment, electromagnetic fields governed by the equations of motion for an electromagnetic field in a dielectric *do* propagate superluminally in the Geroch–Earman sense. Finally, I will argue that an oft-cited argument due to Sommerfeld (1914) intended to show that superluminal wavefront velocities are impossible has nothing to do with relativity *per se*, and instead gains what force it has from an assumption concerning the nature of the interaction between electromagnetic fields and matter motivated by the atomic theory of matter.

Let me also emphasize what I am *not* arguing. I do not mean to argue that there are physical systems that, under realistic conditions, *do* exhibit superluminal propagation in the Geroch–Earman sense. Nor do I mean to argue that it is possible to engineer a dielectric medium through which one could send a signal superluminally, let alone that such media have already been produced. In this way, the title of the paper may be misleading, as I do not mean to argue that the dogma noted above is false. But I do hope to show that we do not understand the relationship between relativity theory and superluminal propagation as well as we might think, even in cases of manifest physical interest (insofar as we have built components of optical systems that exhibit some of the relevant properties). And in this sense, the dogma that relativity theory simply forbids superluminal propagation is unhelpful. On the one hand, it discourages study of *how* relativity theory does and does not accommodate superluminal propagation. And perhaps worse, it may blind us to systems that *do* exhibit superluminal propagation in physically significant and potentially fruitful ways.

The rest of the paper will proceed as follows. I will begin with some preliminaries regarding Maxwell's equations, to establish notation and conventions, and to provide a translation manual between different ways of presenting Maxwell's theory. Next I will reconstruct several standard arguments concerning superluminal propagation of electromagnetic fields in a dielectric. I will then present Geroch's framework for treating the propagation of fields and argue that his approach provides a natural way of precisely recovering the Sommerfeld–Brillouin notion of wavefront velocity. Using this framework, I will analyze the standard relativistic field equations for an electromagnetic field in a dielectric medium and show that for certain parameter values, these fields will exhibit superluminal propagation in the Geroch–Earman sense—i.e., they will have superluminal wavefront velocities. Finally, I will return to Sommerfeld's no-go argument for superluminal wavefront velocities and discuss how the example I present runs afoul of his assumptions. The upshot will be that insofar as Sommerfeld's argument succeeds, relativity theory plays no apparent role. I will conclude by stating, and to some extent responding to, a number of objections to the analysis I give and suggesting avenues for future work.

2. Preliminaries

In what follows, we work in Minkowski spacetime, (M, η_{ab}), in units in which the speed of light, c, equals 1 (though, for emphasis, we will sometimes continue to refer to c as the speed of light).[5] We assume that Minkowski spacetime is endowed with a fixed temporal orientation and a fixed orientation, with associated volume element ϵ_{abcd}. In this context, Maxwell's equations for electromagnetic fields in a vacuum may be written in a compact form as

$$\nabla_a F^a{}_b = J_b \tag{1a}$$

$$\nabla_{[a} F_{bc]} = \mathbf{0}, \tag{1b}$$

where ∇ is the Minkowski spacetime derivative operator, F_{ab} is the electromagnetic field tensor, and J^a is the charge-current density. Indices are raised and lowered with η_{ab}.

The electromagnetic field F_{ab} can be taken to encode the electric and magnetic fields as determined by any observer, as follows. Given an observer with 4-velocity ξ^a at a point p in Minkowski spacetime, the electric field determined by that observer is given by $E^a = F^a{}_b \xi^b$ and the magnetic field is given by $B^a = \frac{1}{2}\epsilon^{abcd}\xi_b F_{cd}$. Similarly, $\sigma = J_a \xi^a$ is the charge density as determined by that observer, whereas $j^a = J^a - (J^n \xi_n)\xi^a$ is the 3-current density determined by that observer.

It will be convenient to be able to move back and forth between this manifestly relativistic form of Maxwell's equations and a more traditional formulation, which is more common in the literature on the propagation of electromagnetic waves.[6] To do so, we will fix, once and for all, a constant future-directed unit timelike vector field ξ^a on Minkowski spacetime, representing, say, the 4-velocities of a family of co-moving inertial observers. Unless otherwise stated, the electric and magnetic fields, E^a and B^a, and the charge and current 3-vector densities, σ and j^a, will always be assumed to be determined relative to this family of observers. The electromagnetic field tensor F_{ab} can be reconstructed in terms of these fields as

$$F_{ab} = 2E_{[a}\xi_{b]} + \epsilon_{abnm}\xi^n B^m. \tag{2}$$

Eqs. (1) then can be re-written as

$$\nabla_{[a} F_{bc]} = 0 \Longleftrightarrow \begin{cases} \partial_b B^b = 0 \\ \epsilon^{abc}\partial_b E_c = -\xi^b \nabla_b B^a \end{cases} \tag{3a}$$

[3] So as not to besmirch their good names, let me emphasize that neither Geroch nor Earman suggests that there are physical systems that *do* propagate superluminally—and indeed, Earman takes the upshot of the discussion to be a more precise characterization of what we intend relativity to prohibit, as a guide to building a prohibition on superluminal propagation into relativistic quantum field theory.

[4] This quantity is often called the "signal velocity" in the literature. It is interesting to note, however, that Sommerfeld himself distinguishes the wavefront velocity he defines from the signal velocity (i.e., the group velocity) that Brillouin discusses (Brillouin, 1960, p. 19). I will follow Sommerfeld and call this quantity the "wavefront velocity".

[5] Minkowski spacetime (M, η_{ab}) is a relativistic spacetime where M is $\mathbb{R}^4$ and η_{ab} is flat and geodesically complete. Throughout we use the "abstract index" notation developed by Penrose & Rindler(1984) and used by Wald (1984) and Malament (2012). We adopt the convention that the Minkowski metric has signature (1,3), so that timelike vectors have positive inner product with themselves.

[6] For further details on the relationship between these formulations, see Malament (2012). When I say "relativistic" in this setting, I mean independent of a choice of observer or coordinate system.