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A branching space-times view on quantum error correction

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Abstract

In this paper we describe some first steps for bringing the framework of branching space-times (BST) to bear on quantum information theory. Our main application is quantum error correction. It is shown that BST offers a new perspective on quantum error correction: as a supplement to the orthodox slogan, "fight entanglement with entanglement", we offer the new slogan, "fight indeterminism with indeterminism".

Keywords: Indeterminism; Quantum information; Histories; Branching space-times

1. Introduction

Branching space-times (BST; Belnap, 1992) is a general formal framework for describing indeterminism in a relativistic setting. We have good reason to assume that our world is both relativistic (the speed of light is the limiting velocity for signal transmission) and indeterministic (the way things are now does not always fix uniquely how things will be in the future). BST gives a formal model of how this could both be true.

Indeterminism means that certain situations allow for more than one possible continuation in their future. In a Newtonian world, this might be spelled out in terms of instantaneous states of the whole world across all of space. Relativity theory appears not to allow for an invariant notion of "instantaneous state of the world". Thus, in the spirit of relativity theory, indeterminism must rather be a local affair. Accordingly, BST pictures basic indeterministic happenings as *basic transitions* that consist of a single point-like event

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and one of its multiple possible causal futures (possible future light cones of the event). More generally, in the framework of BST all indeterministic happenings can be viewed as indeterministic transitions that can be accounted for through the basic indeterministic transitions that are causally involved in them. Along these lines, the theory of transitions in BST has been used to give an analysis of causation (Belnap, 2005) and of single-case probabilities (Belnap, 2007; Müller, 2005; Weiner & Belnap, 2006).

Apart from such general applications, the formal framework of BST can also be used as a background for the formulation of specific physical theories. An obvious candidate is quantum mechanics, since that theory has both an indeterministic aspect, in the form of measurements and their outcomes, and is in need of a spatio-temporal setting, in view of the phenomenon of quantum correlations. A number of studies has shown the usefulness of a BST perspective on quantum mechanics (Belnap & Szabò, 1996; Müller, 2002; Müller & Placek, 2001; Placek, 2000). A recent result (Müller, 2007) furthermore shows how the histories approach to quantum mechanics (Gell-Mann & Hartle, 1990, 1993; Griffiths, 1984, 2003) can be interpreted within a subset of BST, viz., the branching time formalism. Thus there is some hope of arriving at a histories-type formulation of relativistic quantum mechanics based on BST.

One of the fastest growing areas of contemporary physics, with many promising potential applications, is the area of quantum computation and quantum information (cf. Nielsen & Chuang, 2000, for an overview). That field of quantum theory has not yet been approached from a BST perspective. Such an approach, however, appears promising: Firstly, from a general point of view, a quantum computer is an indeterministic spatio-temporal system, and describing such a computer in a formal framework unifying these two basic aspects may prove to be beneficial. Secondly, there are specific issues in quantum information theory in which indeterminism plays an important role and for which the formal tools of BST may provoke fresh insights. Two such fields are the theory of one-way quantum computation (Browne & Briegel, 2006; Raussendorf & Briegel, 2001) and the theory of quantum error correction (Nielsen & Chuang, 2000, Chapters 8–10).

In this paper, we will focus on the latter issue, i.e., quantum error correction, as a first step of bringing together the fields of BST and quantum information. In order for the discussion to be accessible for readers familiar with only one of the sides, our presentation is perhaps more verbose than might be usual, and some readers may wish to skip a few paragraphs introducing material with which they are already familiar. The overall aim of this paper is to advertise methodological pluralism. We do not prove any new theorems or provide new error correcting codes. Nor do we claim that a BST view on quantum information is the only viable position. Rather, we wish to show that a BST perspective is *a* viable position, which may prove to be illuminating for conceptual issues of quantum information in general and quantum error correction in particular.

The paper is structured as follows: Section 2 reviews some key concepts of BST. Section 3 then gives a brief overview of some aspects of quantum information theory and introduces the subject of quantum error correction. Finally, in Section 4 we present our BST based view on quantum error correction.

2. Branching space-times

BST takes seriously the view that our world, the world we live in, is both indeterministic and relativistic. There have long been available formal frameworks for picturing our world