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# Léon Rosenfeld and the challenge of the vanishing momentum in quantum electrodynamics

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ABSTRACT

Léon Rosenfeld published in 1930 the first systematic Hamiltonian approach to Lagrangian models that possess a local gauge symmetry. The application of this formalism to theories with local internal symmetries, such as electromagnetism in interaction with charged matter fields, is valid and complete, and predates by two decades the work by Dirac and Bergmann. Although he provided a group-theoretical justification for gauge fixing procedures that had just been implemented in the first expositions of quantum electrodynamics by Heisenberg and Pauli, and also by Fermi, his contribution went largely unnoticed. This lack of impact seems to be related to a generalized disenchantment with second quantization in the 1930s and 1940s.





## 1. Introduction

Léon Rosenfeld is best known for his treatment with Niels Bohr in 1933 of the measurability of quantum electrodynamic fields (Bohr & Rosenfeld, 1933).[1] Less well-known is his groundbreaking analysis of the phase space implementation of gauge symmetries that he published in *Annalen der Physik* in 1930 under the title "Zur Quantelung der Wellenfelder" (On the Quantization of Wave Fields) (Rosenfeld, 1930a).[2] In this paper I will discuss Rosenfeld's invention of constrained Hamiltonian dynamics, and in particular his application of this formalism to quantum electrodynamics. Ultimately we would like to know why neither Wolfgang Pauli, who had recommended this analysis to Rosenfeld, nor apparently anyone else in the ensuing 20 years, acknowledged the pertinence of this work for the development of quantum electrodynamics. Indeed, it was only following the work of Peter G. Bergmann and P. A. M. Dirac, commencing in 1949,[3] that the systematic treatment of constrained Hamiltonian systems began to attract attention. This formalism is now known as the Dirac–Bergmann procedure. Bergmann's interest was in the Hamiltonian version of Einstein's general theory of relativity, as a first step in its eventual quantization. He was initially not aware of Rosenfeld's work, but when he did learn of it he consistently cited it as a forerunner of his own work. Dirac on the other hand, as we shall see below, was already in 1932 aware of Rosenfeld's formalism, specifically in regard to its application to quantum electrodynamics. Yet as far as I can tell Dirac never acknowledged Rosenfeld's contribution. I do not wish to debate priorities in this paper. Rather, we shall attempt to understand the contextual dynamic of this story with the hope that it will shed light not only on the early development of quantum electrodynamics but also on the subsequent development of gauge theories.

Initial progress with canonical electrodynamic quantum field theory was temporally stymied in 1929 by the identical vanishing of the momentum associated with the temporal component of the electromagnetic potential. I will first briefly review the earlier history of quantum electrodynamics, then discuss the not altogether satisfactory resolution of this quandry that was published by Pauli and Werner Heisenberg. This is where Rosenfeld enters the stage. Following a brief biographical sketch I will then review his pioneering constrained dynamics formalism with a description in detail of his application of the program to Lorentz covariant electrodynamics. Then I will address the resounding lack of impact of his contribution. An Appendix

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[1] An English translation appears in Bohr and Rosenfeld (1979).

[2] An English translation of this paper, with extensive annotation, will appear as a Max Planck Institute for the History of Science preprint. A critical analysis will appear in *Archive for History of Exact Sciences*.

[3] Both Dirac (1950, 1951) and Bergmann and his collaborators (Anderson & Bergmann, 1951; Bergmann, 1949; Bergmann & Brunings, 1949; Bergmann, Penfield, Schiller, & Zatkis, 1950) laid out the general formalism in the period from 1949 to 1951.

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contains a group theoretical discussion of the canonical imposition of gauge conditions.

## 2. Quantum electrodynamics before 1930

The understanding of the interaction between electrically charged matter and the electromagnetic field was of course a focus of the emerging theory of quantum mechanics from the very beginning of its history. Pais (1982) has argued that quantum mechanics and quantum electrodynamics both found their origins in Planck's black body energy density formula of 1900 (Planck, 1900), though the dates and individuals he attributes to these beginnings might surprise. He would have Einstein in 1906 as the first to have quantized the material oscillator.[4] Debye (1910) in 1910 is the first to have quantized the free radiation field—and he thereby derived the Planck formula. He achieved this result by treating each oscillation mode as an independent quantized harmonic oscillator. However, there was substantial resistance to the idea of quantizing the radiation field. The 1924 Bohr Kramers Slater program (Bohr, Kramers, & Slater, 1924) actually represents the final failed effort within the framework of the old quantum theory to confine quantum effects to the electromagnetic interaction with charged matter; the electromagnetic field itself was thought to remain subject to the classical Maxwell description. That the photon particle actually existed was then demonstrated irrefutably first by Bothe and Geiger (1925) and then by Compton and Slater (1925). The former work already led Bohr to observe that "Under these circumstances we must be prepared for the fact that the generalization of classical electrodynamic theory that we are seeking will require a thoroughgoing revolution of the concepts on which the description of nature has until now been based."[5]

This revolution followed in very short order, in step with the emergence of the new quantum theory. Indeed, its creators almost invariably sought to broaden the scope of new technical and conceptual insights to include electromagnetic interactions. Often they did this in their original groundbreaking papers. So, for example, in Born and Jordan (1925), following Heisenberg's lead, proposed that the electric and magnetic fields ought to be represented by matrices. This suggestion preceded the epochal deduction of the position and momentum commutation relation for finite dimensional systems in their joint paper the following year with Heisenberg (Born, Jordan, & Heisenberg, 1926), and they did not inquire into the electromagnetic field algebra. Jordan's contribution in the latter paper is often cited as the beginning of quantum field theory. He introduced canonical commutation relations for the Fourier modes of a field theory with one spatial dimension: a string. He was able to calculate the mean squared energy fluctuations for this theory, obtaining a sum of two expressions—one of which was clearly of particle origin and the other clearly the result of wave interference. Such an expression had originally been obtained by Einstein (1909) from Planck's energy density formula by applying statistical mechanical arguments. We witness here an instance in which the obtainment of a desired quantum field theoretical result actually buttressed the belief of researchers that they were making progress in formulating a correct theory of quantum mechanics for finite systems.

It was Dirac who first saw the relation between the commutation rule $qp - pq = i\hbar$ and Poisson brackets, thus creating a general algebraic canonical quantization rule, a rule that did not necessitate the use of matrices (Dirac, 1925). He recounts in his 1977 Varenna lectures that the idea came to him "in a flash".[6] Hamiltonian dynamics was not at that time a staple in the education of a young physicist, but he had already used it extensively. The classical canonical transformation formalism suggested to him a quantum mechanical analogue that he dubbed "transformation theory". In modern parlance one important aspect of Dirac's theory is that it offers a freedom to change representations. It would therefore serve as a basis for the demonstration of the equivalence of the Heisenberg–Born–Jordan matrix representation and Schrödinger's wave mechanics, to be addressed shortly. But perhaps the most important aspect of Dirac's transformation theory for this essay is that it provides a means for translating classical canonically implemented symmetries as transformations of quantum variables. Strangely, in a 1972 historical talk on the occasion of Dirac's seventieth birthday, Jost (1972), chap. 6 also notes that "Dirac's deep affinity for analytical dynamics is still noticeable in his papers on quantum electrodynamics ...". But then he goes on to remark that "... this is a use that we would hardly find justified in our own time". This is consistent with the idea that widespread interest in canonically implemented gauge symmetries really grew only after 't Hooft's (1971) proof of the renormalizability of non-Abelian gauge theories. The thesis is supported in the same volume by Lanczos (1972), who notes that almost no one involved in the development of quantum mechanics at this time recognized the group-theoretical significance of Dirac's quantum transformation theory.

As he recounted in his Varenna lectures, Dirac's dominant interest early in his graduate career was in relativity theory. Combined with his expertise in Hamiltonian dynamics it was natural for him to work out independently a Hamiltonian formulation of charged particles in interaction with an external electromagnetic field. He writes, "When I first met this problem, I proceeded to solve it without bothering to look up the literature to see whether it had been solved previously ... [It] did not involve much difficulty, and I think it was much simpler than looking up the references."[7] I quote this in part to highlight a recurrent feature of Dirac's work, his tendency to follow up on hunches without undertaking extensive literature searches. Dirac's first foray into relativistic Hamiltonian dynamics resulted in a remarkable treatment of the Compton effect, employing action and angle variables in the context of the old quantum theory (Dirac, 1926). The work is innovative in two respects. First, he managed to get the desired result while still treating the radiation field as a classical external field. We find here also very likely the first appearance of a quantum Hamiltonian constraint: he promotes the time to a quantum operator with the consequence that the Hamiltonian vanishes. It was this work that occasioned the following interchange with Thomas Kuhn in a 1963 *Archive for the History of Science* Interview[8]

> Kuhn: You develop it classically first and then simply apply commutation relations to W and t; the classical formulation is one that I hadn't seen ...
>
> Dirac: I think it is rather standard that you can count time as an extra variable and introduce something conjugate to it.
>
> Kuhn: Do you think it was relatively standard at the time? I don't know of another place where this point had been put previously in this way, but I'm not at all sure it hadn't.

[4] Pais (1982, p. 378).

[5] "Bei dieser Sachlage muss man darauf vorbereitet sein, dass die zu erstrebende Verallgemeinerung der klassischen elektrodynamischen Theorie eine durchgreifende Revolution der Begriffe fordert, auf denen die Naturbeschreibung bis jetzt beruht hat" (Bohr, 1925, p. 155).

[6] Dirac (1977), p. 122.

[7] Dirac (1977), p. 143.

[8] AHQM, May 10, 1963, p. 15.