Chemometrics and Intelligent Laboratory Systems 152 (2016) 34–41

Contents lists available at ScienceDirect

Chemometrics and Intelligent Laboratory Systems

journal homepage: www.elsevier.com/locate/chemolab

Design of fractional order predictive functional control for fractional industrial processes

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ARTICLE INFO

Article history:
Received 18 November 2015
Received in revised form 13 January 2016
Accepted 14 January 2016
Available online 22 January 2016

Keywords:
Fractional order calculus
Fractional order system
Predictive functional control
Oustaloup approximation
Integer reduced order model

ABSTRACT

Many phenomena in practical processes cannot be accurately described by conventional differential equations, while fractional order differential equations can describe the characteristics of such processes more accurately. In this paper, the fractional order predictive functional control (FPFC) method is designed for a class of single-input single-output (SISO) fractional order linear systems. The Oustaloup approximation is employed to derive the approximate model of fractional order system. Meanwhile, the Grünwald–Letnikov (GL) definition and the fractional calculus operator are used in its cost function, which further extend the applications of fractional order calculus to the predictive functional control algorithm. And then the optimal control is obtained. Compared with traditional predictive functional control based on integer reduced order model, simulation results reveal that the fractional order controller yields improved control performance.

1. Introduction

The theory of fractional order calculus can be traced back to 300 years ago, but its application in the field of control engineering is nearly 20 years [1]. This study extends the use of calculus to not only integer order but also fractional order systems. One of the reasons maybe that a lot of phenomena cannot be accurately described by the integer order differential equations in realistic physical systems and the fractional order differential equation models can be more accurate than traditional integer order models to express the characteristics of the actual systems [2]. For this purpose, fractional order calculus has been used to model the complex industrial processes by several researchers [3–5]. Meanwhile, in [6], the recursive least-square method and recursive instrumental variable algorithm were used to estimate the parameters of fractional order models with generalized ARX structure. Madakyaru et al. proposed an approach to reconstruct the ARX models using the fractional order differential operators and orthonormal basis filters. In their paper, the models were identified from input–output perturbation data using a two-step nested optimization scheme and the experimental studies on practical benchmark heater-mixer setup shows the feasibility of this method [7].

In the field of control systems, the PD^{μ}controller [8], CRONE controller [9] and $PI^{\lambda}D^{\mu}$ controller [10] were proposed sequentially in the 20th century. With the wide use of PID controllers in industries, more and more studies involving fractional order calculus have been done to explore better design methods for fractional order control systems in recent years. For example, Yeroglu and Tan [11] presented the design techniques of fractional order PID controllers, and the Ziegler–Nichols method and Åström–Hägglund method were applied for the tuning of their controller parameters. In [12], a genetic algorithm was introduced to improve the accuracy of the designed fractional order $PI^{\lambda}D^{\mu}$ controller. A tuning graphical method of fractional order PID controller was studied in [13] and a tuning graphical method for fractional order $PI^{\lambda}D^{\mu}$ controllers was proposed on the basis of the sensitivity function constraint of the closed-loop transfer function in [14]. Jin et al. proposed a model reduction method and an explicit PID tuning rule for the PID auto-tuning based on fractional order system in [15]. In [16], the authors clearly came to the conclusion that a fractional order controller can obtain better performance than integer order controller for fractional order systems. As a consequence, a study of the fractional order control theory and its application in the field of practical process control is of great significance.

On the other hand, the motivation of the research on fractional order calculus is that fractional order models can fit the actual data more precisely and flexibly than integer order models and the outstanding merit of the fractional order model has laid a good foundation for model predictive control (MPC) based on process models. Predictive functional control (PFC) is one type of model predictive control technologies and it has been proposed by Richalet to control the dynamic system [17]. In this strategy, the control input is derived by solving the difference between the future predicted output and the desired trajectory by minimizing the cost function. PFC is the most popular one which has been widely used in research and practical control engineering that also

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http://dx.doi.org/10.1016/j.chemolab.2016.01.011
0169-7439/

reveal good control effect [18,19]. In [20], the online recursive least-squares identification and the self-adaptive predictive functional control algorithm were applied to the temperature control system. For complex nonlinear systems, the PFC methods based on fuzzy models were presented by lots of scholars [21–23]. Also, a predictive functional controller based on extended state space model was developed and it shows good control performance when it was compared with traditional state space PFC in [24,25]. The extended state space process predictive functional control algorithm based on genetic algorithm optimization is designed for batch processes under actuator faults in [26]. Based on the extended state space PFC method, Tao et al. designed a controller with the linear quadratic form to deal with the process under partial actuator failures in [27]. The extended state space PFC algorithms based on decoupling strategy that uses the adjugate matrix of the process [28] or partially decoupled scheme [29] have also been used for multivariable processes. In addition, based on the optimization idea, PFC was used to optimize the PI or PID controllers to obtain the advantages of both PFC and PID [30–32]. Though successful in process control, most PFC strategies are for integer order systems.

At present, several studies on the combination of fractional order calculus and MPC algorithm have been applied prosperously. The predictive functional controller based on state space model has been presented for fractional order systems and the two basic functions were considered in [33]. In [34], the GL definition was used to discretize the fractional order system and the fractional integral was considered in the cost function. The excellent performance of the designed non-minimal state space fractional order predictive functional controller for fractional order systems was displayed on the thermal fractional system. In [35], the adaptive generalized predictive control (GPC) algorithm was designed for fractional order dynamic model of solid oxide fuel cells. In [36–37], Romero et al. proposed a new GPC algorithm for fractional order systems and the fractional order operators were used in its cost function. The numerical approximation model and Oustaloup approximation model were used to predict the future dynamic output of the system, and the proposed MPC for fractional order control system achieved satisfactory performance by Rhouma et al. in [38–39]. Based on time domain, Guo et al. proposed some new control methods which combine the virtues of the fractional order PID algorithm and predictive control algorithm [40–42]. Moreover, Joshi et al. proposed an MPC method that can track the reference signals with limited uncertainties for fractional order systems. In particular, the Laplace transform of Caputo fractional order calculus and Mittag–Leffler function (MLF) were used to evaluate the process output. However, MPC was employed in a fractional order system with the fractional order α with $0<\alpha<1$ [43]. As shown above, the research on PFC for fractional order systems is limited in quantity and there are still requirements for new methods to achieve better performance of fractional order predictive controllers.

The main aim of our study is to develop a new approach to control the fractional order system and further improve the performance of the control system when compared with the integer order PFC since the industrial processes are fractional order systems in essence. In this paper, the proposed controller has been designed for the system described by SISO linear fractional differential equations. In addition, the fractional order derivative in the cost function of fractional PFC (FPFC) algorithm is expected to enhance the performance because of more tuning parameters. The proposed FPFC algorithm design is as follows. First, the input–output process model has been derived from the Oustaloup approximation of fractional order transfer function. Second, the predicted output is transformed into the matrix-form prediction. Then, the GL definition will be utilized to discrete the fractional order cost function. This method has such features as simple calculation, strong robustness, and strong anti-interference ability. Finally, FPFC shows good performance when it is successfully employed to practical heating furnace process.

The paper is organized as follows. In Section 2, the basic knowledge of fractional order calculus is described. In Section 3, the design of FPFC for fractional order systems is presented. Firstly, we use Oustaloup approximation method to approximate fractional order operator s^{α}, and the model of the fractional controlled process is obtained. Then, based on the obtained model, the integer order PFC is extended to the non-integer order predictive functional control to get the optimal control law. In Section 4, some simulation results are done to verify the performance of the FPFC controller on a heating furnace. Conclusion of the proposed method is drawn in Section 5.

2. Fractional order calculus

Fractional order calculus is expanded from traditional calculus, which allows the differential and integral equations to be of fractional orders instead of integer orders. With the rapid development of the theory of fractional calculus, several definitions available for fractional calculus are defined consecutively. There is no single definitions of fractional calculus so far. The three commonly used definitions are Grünwald–Letnikov (GL), Riemann–Liouville (RL), and Caputo definitions [44].

The GL definition of fractional order calculus is described as follows:

$$ {}_aD_t^{\beta} f(t) = \lim_{h\to 0} \frac{1}{h^{\beta}} \sum_{j=0}^{[(t-a)/h]} (-1)^j \binom{\beta}{j} f(t-jh) \tag{1}$$

where, a is the initial time, h is the selected appropriate calculation step, $[(t-a)/h]$ is the integer part of the number $(t-a)/h$,$\omega_j^{(\beta)} = (-1)^j \binom{\beta}{j}$ is the polynomial coefficients and can be solved as

$$ \omega_0^{(\beta)} = 1\ ,\ \ \omega_j^{(\beta)} = \left(1-\frac{\beta+1}{j}\right)\omega_{j-1}^{(\beta)},\ \ j = 1,2,\cdots. \tag{2}$$

Considering the practical process and the short-term memory characteristics of fractional order calculus operator, the sample time T_s is substituted for calculating step h. For simplicity, we denote $D^{\beta} \equiv {}_aD_t^{\beta}$ with zero initial conditions, Eq. (1) can be converted into the following form:

$$ D^{\beta} f(t) = \frac{1}{T_s^{\beta}} \sum_{j=0}^{n} (-1)^j \binom{\beta}{j} f((n-j)T_s) \tag{3}$$

where, $n=[(t-1)/T_s]$.

The RL definition of fractional order calculus can be defined as

$$ D^{\beta} f(t) = \frac{1}{\Gamma(m-\beta)} \frac{\mathrm{d}^m}{\mathrm{d}t^m} \int_a^t \frac{f(\tau)}{(t-\tau)^{\beta-1}} \mathrm{d}\tau \tag{4}$$

where, $m-1<\beta<m,\ m\in \mathrm{N}$, and $\Gamma(\cdot)$ is the Euler's gamma function:

$$ \Gamma(x) = \int_0^{\infty} e^{-t} t^{x-1} dt \tag{5}$$

The Caputo definition of fractional order calculus is described as

$$ D^{\beta} f(t) = \frac{1}{\Gamma(m-\beta)} \int_a^t \frac{f^m(\tau)}{(t-\tau)^{1+\beta-m}} \mathrm{d}\tau \tag{6}$$

The Laplace transform of RL fractional calculus is:

$$ \mathrm{L}\left\{D^{\beta} f(t)\right\} = s^{\beta} f(s) - \sum_{k=0}^{m-1} s^k D^{\beta-k-1} f(t)|_{t=0^+} \tag{7}$$