Chemometrics and Intelligent Laboratory Systems 142 (2015) 9–17

Contents lists available at ScienceDirect

Chemometrics and Intelligent Laboratory Systems

journal homepage: www.elsevier.com/locate/chemolab

# Fault isolation of non-Gaussian processes based on reconstruction

Yunpeng Fan, Zhengbing Wang, Yingwei Zhang *, Hong Wang

*State Key Laboratory of Synthetical Automation for Process Industries, Northeastern University, Liaoning 100819, PR China*

ARTICLE INFO

*Article history:*
Received 8 May 2014
Received in revised form 26 December 2014
Accepted 30 December 2014
Available online 12 January 2015

*Keywords:*
Non-Gaussian processes
Fault isolation
Fault relevant direction

ABSTRACT

Classical fault detection method has been successfully applied to practical industrial processes. However, fault isolation is still a difficult issue yet to be solved. It is obvious because the existing fault isolation methods have not fully used the embedded information of the concerned faults, where accuracy and reliability of fault isolation cannot be guaranteed. For this purpose this paper presents a novel data driven fault isolation approach for non-Gaussian processes. The proposed method firstly utilizes an offline learning mechanism to obtain information on normal operating conditions together with the learning on the variance and covariance of the faults. And then it is followed by the retrieving of the fault relevant directions and the construction of detection statistics together with the relevant confidence limit. As such, the fault identification model based on reconstruction for fault and residual subspace is obtained which represents healthy operation status of the process. In this way an effective data driven fault isolation algorithm is established. In addition, the proposed method has been applied to the fault isolation for an electro-fused magnesia furnace (EFMF) and desired results have been obtained.


## 1. Introduction

In order to ensure the safety of process operation and product quality, the batch process monitoring has become a key issue for investigation [1]. Especially for the system with multiple variables and strong correlation, traditional monitoring methods are unable to achieve the required accuracy. As such, multivariate statistical process monitoring (MSPM) [2–4] methods have been proposed and exhibited great success in the monitoring of industrial processes [33].

Over the past few decades, some effective MSPM methods have been widely used for fault diagnosis. Examples are principal component analysis (PCA) [6] and partial least squares (PLS) [7]. In this context, the conventional fault diagnostic methods based on PCA extract the underlying information from sampled data and define an acceptable operating region called confidence limit by some rules. If the monitoring indexes move outside the confidence limit, it indicates an unusual change or a process fault has occurred. Compared to PCA, PLS pays much attention to the quality of the output as the fault detection is not sufficient for industrial production. When a fault is detected, it is always hoped that effective fault isolation can be realized so as to quickly find the source of the fault. In recent years, fault isolation techniques based on PCA have been proposed. In the multivariate statistical process monitoring area, the method of contribution plots [4,15] has been widely used for fault isolation under the assumption that the variables with the largest contributions to the fault detection index are most likely to be the faulty variables. However, this method may result in confusing results because of the coupling among the concerned process variables. To solve this problem, the concept of fault reconstruction was defined in the work of Dunia and Qin [5]. Also, reconstruction-based contribution (RBC) [4] was developed, where the reconstruction of a fault detection index was defined along the direction of a variable as the variables' contribution for fault diagnosis. Since most industrial processes have nonlinear characteristics, kernel principal component analysis (KPCA) [18–22] was proposed and widely used recently as a nonlinear extension of linear PCA [23–28]. It has been shown that KPCA can efficiently calculate principal components in a high-dimensional feature space [29].

Considering that the operation faults are the inherent nature of many processes and each fault exhibits significantly different underlying behaviors, it is difficult to develop multiple models for the fault isolation [30]. Then each model represents a specific phase and explains the local process behaviors, which can effectively enhance process understanding and improve monitoring reliability. MPCA models were used to analyze the nature of a two-phase jacketed exothermic batch chemical reactor, where monitoring results show that the two phase-based models are more powerful than a single model [31]. Lu et al. developed a clustering-based division algorithm and phase-based sub-PCA modeling method. The method is based on the fact that changes of the process correlations may relate to the fault phase shift in the batch processes. The improved PCA method is available for the Gaussian processes. However, it performs poorly when it is applied to industrial process data with non-Gaussianity [8,9]. Yu et al. developed a particle filter based dynamic Gaussian mixture model (DGMM) [10]. More recently, independent component analysis (ICA) [11–14] was introduced to solve this problem. ICA provides much meaningful statistical analysis because ICA assumes that the latent variables are not

* Corresponding author.
*E-mail address:* zhangyingwei@mail.neu.edu.cn (Y. Zhang).

http://dx.doi.org/10.1016/j.chemolab.2014.12.013

Gaussian distributed, which involves higher-order statistics, that is, it reduces higher order statistical dependencies compared to PCA. Hence, independent components (ICs) reveal more useful information on higher-order statistics from observed data than principal components (PCs) [16,17].

Therefore in this paper a novel data driven fault isolation algorithm using reconstruction techniques is developed for industrial processes, where the main contributions of this paper are listed as follows:

(1) The proposed method has eliminated the confusing results on fault isolation seen in the existing approaches.
(2) The fault-relevant directions according to their interference degree to the monitoring indexes are extracted so that the relationship between normal status and fault case can be analyzed in details.
(3) The historical data of faults is used to build the fault directions.

The rest of this paper is organized as follows: The fault reconstruction and isolation method based on fault-relevant kernel independent components are proposed in Section 2. The simulation results of the EFMF process are presented in Section 3. Finally, the conclusion is given in Section 4.

## 2. Fault reconstruction and isolation method

### 2.1. Extracting fault-relevant independent components

In this section, the fault-relevant kernel independent components are described. In kernel independent component analysis (KICA), the training data sets $\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_n \in R^J$ from the normal process and $\mathbf{x}_{f,1}, \mathbf{x}_{f,2}, \cdots, \mathbf{x}_{f,n} \in R^J$ from the fault space are mapped into the feature space respectively. The data sets in feature space are recorded as $\Phi(\mathbf{X}) = [\Phi(\mathbf{x}_1), \Phi(\mathbf{x}_2), \cdots, \Phi(\mathbf{x}_n)]$ and $\Phi(\mathbf{X}_f) = [\Phi(\mathbf{x}_{f,1}), \Phi(\mathbf{x}_{f,2}), \cdots, \Phi(\mathbf{x}_{f,n})]$.

To obtain the principal components in the feature space, one needs to solve the equation

$$\lambda_l \boldsymbol{\gamma}^l = \frac{1}{n}\mathbf{K}\boldsymbol{\gamma}^l \tag{1}$$

where matrix $\mathbf{K}$ is defined as $[\mathbf{K}]_{i,j} = \langle \Phi(\mathbf{x}_i), \Phi(\mathbf{x}_j)\rangle$, and it is calculated by $K_{i,j} = \exp(-\|\mathbf{x}_i - \mathbf{x}_j\|^2/c)$ in this paper. $\lambda_l$ denotes the $l$-th eigenvalue of $\mathbf{K}$ in the feature space and $\boldsymbol{\gamma}^l$ is the corresponding eigenvector which can be recorded as $\boldsymbol{\gamma}^l = [\gamma_1^l, \gamma_2^l, \cdots, \gamma_n^l]^T$.

To satisfy the assumption that $\sum_{k=1}^{n}\Phi(\mathbf{x}_k) = 0$, the kernel matrix $\mathbf{K}$ should be mean-centered before calculation by $\mathbf{K} = \mathbf{K} - \mathbf{KI} - \mathbf{IK} + \mathbf{IKI}$, where each element of $\mathbf{I} \in R^{n\times n}$ is equal to $1/n$. The eigenvector $\boldsymbol{\gamma}^l$ should be also normalized to satisfy $\|\boldsymbol{\gamma}^l\|^2 = 1/(n\lambda_l)$.

In the feature space, the data sets $\Phi(\mathbf{X})$ and $\Phi(\mathbf{X}_f)$ are separated into the systematic subspace and the residual subspace, respectively. $\mathbf{P}$ with $R$ directions span the systematic subspace of $\Phi(\mathbf{X})$, and the $l$-th column of $\mathbf{P}$ is given by $\mathbf{p}^l = \sum_{i=1}^{n}\gamma_i^l\Phi(\mathbf{x}_i) = \Phi(\mathbf{X})\boldsymbol{\gamma}^l$. Therefore, there is $\mathbf{P} = \Phi(\mathbf{X})\boldsymbol{\Gamma}$, where $\boldsymbol{\Gamma} = [\boldsymbol{\gamma}^1, \cdots, \boldsymbol{\gamma}^l, \cdots, \boldsymbol{\gamma}^R]$. Similarly, $\mathbf{P}^*$ with the remain $R^*$ directions can be recorded as $\mathbf{P}^* = \Phi(\mathbf{X})\boldsymbol{\Gamma}^*$. Moreover, $\mathbf{P}_f$ and $\mathbf{P}_f^*$ which span the systematic subspace and the residual subspace of $\Phi(\mathbf{X}_f)$ can also be denoted as $\begin{cases}\mathbf{P}_f = \Phi(\mathbf{X}_f)\boldsymbol{\Gamma}_f \\ \mathbf{P}_f^* = \Phi(\mathbf{X}_f)\boldsymbol{\Gamma}_f^*\end{cases}$.

$\mathbf{P}$ can be modified as

$$\mathbf{W} = \mathbf{P}\mathbf{D}^{-1/2}\mathbf{B}\mathbf{D}^{1/2} \tag{2}$$

where $\mathbf{D} = diag\{\lambda_1, \lambda_2, \cdots, \lambda_R\}$. $\mathbf{B}$ can be calculated in the feature space using the computation steps of the original ICA.

Let $\mathbf{A} = \boldsymbol{\Gamma}\mathbf{D}^{-1/2}\mathbf{B}\mathbf{D}^{1/2}$, the linear operator $\mathbf{W}$ in the feature space which can recover the ICs from $\Phi(\mathbf{X})$ can be simplified to give:

$$\mathbf{W} = \Phi(\mathbf{X})\boldsymbol{\Gamma}\mathbf{D}^{-1/2}\mathbf{B}\mathbf{D}^{1/2} = \Phi(\mathbf{X})\mathbf{A}. \tag{3}$$

Similarly, there are

$$\begin{cases}\mathbf{W}^* = \Phi(\mathbf{X})\boldsymbol{\Gamma}^*\mathbf{D}^{*-1/2}\mathbf{B}^*\mathbf{D}^{*1/2} = \Phi(\mathbf{X})\mathbf{A}^* \\ \mathbf{W}_f = \Phi\left(\mathbf{X}_f\right)\boldsymbol{\Gamma}_f\mathbf{D}_f^{-1/2}\mathbf{B}_f\mathbf{D}_f^{1/2} = \Phi\left(\mathbf{X}_f\right)\mathbf{A}_f \\ \mathbf{W}_f^* = \Phi\left(\mathbf{X}_f\right)\boldsymbol{\Gamma}_f^*\mathbf{D}_f^{*-1/2}\mathbf{B}_f^*\mathbf{D}_f^{*1/2} = \Phi\left(\mathbf{X}_f\right)\mathbf{A}_f^*\end{cases}. \tag{4}$$

### 2.2. Extracting the fault-relevant directions

To establish the relationship between normal status and fault case and describe the normal range in the fault space, the normal systematic subspace $\Phi^T(\mathbf{X})\mathbf{W}\mathbf{W}^T$ should be projected into the fault systematic subspace and residual subspace as follows

$$\begin{aligned}\hat{\Phi}^T\left(\mathbf{X}_{nf}\right) &= \Phi^T(\mathbf{X})\mathbf{W}\mathbf{W}^T\mathbf{W}_f\mathbf{W}_f^T \\ &= \Phi^T(\mathbf{X})\Phi(\mathbf{X})\mathbf{A}\mathbf{A}^T\Phi^T(\mathbf{X})\Phi\left(\mathbf{X}_f\right)\mathbf{A}_f\mathbf{A}_f^T\Phi^T\left(\mathbf{X}_f\right) \\ &= \mathbf{K}\mathbf{A}\mathbf{A}^T\mathbf{K}_m\mathbf{A}_f\mathbf{A}_f^T\Phi^T\left(\mathbf{X}_f\right)\end{aligned} \tag{5}$$

$$\begin{aligned}\hat{\Phi}^T\left(\mathbf{X}_{nf}^*\right) &= \Phi^T(\mathbf{X})\mathbf{W}\mathbf{W}^T\mathbf{W}_f^*\mathbf{W}_f^{*T} \\ &= \Phi^T(\mathbf{X})\Phi(\mathbf{X})\mathbf{A}\mathbf{A}^T\Phi^T(\mathbf{X})\Phi\left(\mathbf{X}_f\right)\mathbf{A}_f^*\mathbf{A}_f^{*T}\Phi^T\left(\mathbf{X}_f\right) \\ &= \mathbf{K}\mathbf{A}\mathbf{A}^T\mathbf{K}_m\mathbf{A}_f^*\mathbf{A}_f^{*T}\Phi^T\left(\mathbf{X}_f\right)\end{aligned} \tag{6}$$

where matrix $\mathbf{K}_m$ is defined by $[\mathbf{K}_m]_{i,j} = \langle \Phi(\mathbf{x}_i), \Phi(\mathbf{x}_{f,j})\rangle$.

From Eqs. (5) and (6), the kernel matrixes of $\hat{\Phi}(\mathbf{X}_{nf})$ and $\hat{\Phi}\left(\mathbf{X}_{nf}^*\right)$ can be computed as follows:

$$\mathbf{K}_{nf} = \hat{\Phi}^T\left(\mathbf{X}_{nf}\right)\hat{\Phi}\left(\mathbf{X}_{nf}\right) = \mathbf{K}\mathbf{A}\mathbf{A}^T\mathbf{K}_m\mathbf{A}_f\mathbf{A}_f^T\mathbf{K}_f\mathbf{A}_f\mathbf{A}_f^T\mathbf{K}_m^T\mathbf{A}\mathbf{A}^T\mathbf{K}^T \tag{7}$$

$$\mathbf{K}_{nf}^* = \hat{\Phi}^T\left(\mathbf{X}_{nf}^*\right)\hat{\Phi}\left(\mathbf{X}_{nf}^*\right) = \mathbf{K}\mathbf{A}\mathbf{A}^T\mathbf{K}_m\mathbf{A}_f^*\mathbf{A}_f^{*T}\mathbf{K}_f\mathbf{A}_f^*\mathbf{A}_f^{*T}\mathbf{K}_m^T\mathbf{A}\mathbf{A}^T\mathbf{K}^T \tag{8}$$

where matrix $\mathbf{K}_f$ is defined by $[\mathbf{K}_f]_{i,j} = \langle \Phi(\mathbf{x}_{f,i}), \Phi(\mathbf{x}_{f,j})\rangle$.

Let $\mathbf{W}_r$ and $\mathbf{W}_r^*$ be the linear operators that can extract the independent components from $\hat{\Phi}(\mathbf{X}_{nf})$ and $\hat{\Phi}\left(\mathbf{X}_{nf}^*\right)$ respectively. Then it can be obtained that

$$\begin{cases}\mathbf{W}_r = \hat{\Phi}\left(\mathbf{X}_{nf}\right)\mathbf{A}_r \\ \mathbf{W}_r^* = \hat{\Phi}\left(\mathbf{X}_{nf}^*\right)\mathbf{A}_r^*\end{cases}. \tag{9}$$

The scores $\mathbf{Y}_r$ and $\mathbf{Y}_r^*$ can be obtained as follows:

$$\begin{cases}\mathbf{Y}_r = \hat{\Phi}^T\left(\mathbf{X}_{nf}\right)\mathbf{W}_r = \mathbf{K}_{nf}\mathbf{A}_r \\ \mathbf{Y}_r^* = \hat{\Phi}^T\left(\mathbf{X}_{nf}^*\right)\mathbf{W}_r^* = \mathbf{K}_{nf}^*\mathbf{A}_r^*\end{cases}. \tag{10}$$

Projecting $\hat{\Phi}(\mathbf{X}_f)$ on $\mathbf{W}_r$ and $\widetilde{\Phi}(\mathbf{X}_f)$ on $\mathbf{W}_r^*$ respectively, it can be shown that

$$\mathbf{Y}_{fr} = \hat{\Phi}\left(\mathbf{X}_f\right)\mathbf{W}_r = \mathbf{K}_f\mathbf{A}_f\mathbf{A}_f^T\mathbf{K}_f\mathbf{A}_f\mathbf{A}_f^T\mathbf{K}_m^T\mathbf{A}\mathbf{A}^T\mathbf{K}^T\mathbf{A}_r \tag{11}$$

$$\mathbf{Y}_{fr}^* = \widetilde{\Phi}\left(\mathbf{X}_f\right)\mathbf{W}_r^* = \mathbf{K}_f\mathbf{A}_f^*\mathbf{A}_f^{*T}\mathbf{K}_f\mathbf{A}_f\mathbf{A}_f^T\mathbf{K}_m^T\mathbf{A}\mathbf{A}^T\mathbf{K}^T\mathbf{A}_r^* \tag{12}$$

where it has been denoted that

$$\hat{\Phi}^T\left(\mathbf{X}_f\right) = \Phi^T\left(\mathbf{X}_f\right)\mathbf{W}_f\mathbf{W}_f^T = \mathbf{K}_f\mathbf{A}_f\mathbf{A}_f^T\Phi^T\left(\mathbf{X}_f\right) \tag{13}$$

$$\widetilde{\Phi}^T\left(\mathbf{X}_f\right) = \Phi^T\left(\mathbf{X}_f\right)\mathbf{W}_f\mathbf{W}_f^T = \mathbf{K}_f\mathbf{A}_f^*\mathbf{A}_f^{*T}\Phi^T\left(\mathbf{X}_f\right). \tag{14}$$