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Mass measurements and the bound-electron *g* factor

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Dedicated to Professor H.-Jürgen Kluge on the occasion of the 65th birthday.

Abstract

The accurate determination of atomic masses and the high-precision measurement of the bound-electron *g* factor are prerequisites for the determination of the electron mass, which is one of the fundamental constants of nature. In the 2002 CODATA adjustment [P.J. Mohr, B.N. Taylor, Rev. Mod. Phys. 77 (2005) 1], the values of the electron mass and the electron–proton mass ratio are mainly based on *g* factor measurements in combination with atomic mass measurements. In this paper, we briefly discuss the prospects for obtaining other fundamental information from bound-electron *g* factor measurements, we present some details of a recent investigation of two-loop binding corrections to the *g* factor, and we also investigate the radiative corrections in the limit of highly excited Rydberg *S* states with a long lifetime, where the *g* factor might be explored using a double resonance experiment.

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1. Introduction

The central equation for the determination of the electron mass m_e from g factor measurements reads

$$m_e = \frac{\omega_c}{\omega_L} \frac{g|e|}{2q} m_{\text{ion}}, \tag{1}$$

where ω_c is the cyclotron frequency of the ion; ω_L, the Larmor spin precession frequency; q, the ion charge; and m_{ion} its mass. The quantity $e = -|e|$ is the elementary charge, and g is the bound-electron g factor. In most practical applications, the ion is hydrogen like, and the frequency ratio ω_c/ω_L can be determined very accurately in a Penning trap [1,2].

Eq. (1) may now be interpreted in different ways:

- The ratio m_e/m_{ion} is immediately accessible, provided we assume that quantum electrodynamic theory holds for g. Provided the ratio m_{ion}/m_p (with the proton mass m_p) is also available to sufficient accuracy, the electron to proton mass ratio m_e/m_p can be determined by multiplication $m_e/m_{\text{ion}} \times m_{\text{ion}}/m_p$. In the recent CODATA adjustment [3], the ratio m_e/m_p has been determined using two measurements involving ^{12}C.
- Let us suppose that m_{ion} is known to sufficient accuracy. Assuming that quantum electrodynamic theory holds for g, we may then determine m_e from the measurement [4–6].
- The g factor depends on the reduced mass of the electron-ion two-particle system. An accurate measurement of g can therefore yield an independent verification of the isotopic nuclear mass difference, provided that the masses of the ions have been determined beforehand to sufficient accuracy [7].
- Direct access to the electron g factor in a weak external magnetic field depends on the property of the nucleus having zero spin. According to a relatively recent proposal [8,9], the measurement of a g factor for a nucleus with non-zero spin can

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be used to infer the nuclear g factor, provided the purely electronic part of the g factor is known to sufficient accuracy from other measurements.

- There is also a proposal for measuring g factors in lithiumlike systems, and theoretical work in this direction has been undertaken [10]. Provided the contribution due to electron–electron correlation can be tackled to sufficient accuracy, a measurement of the g factor in lithiumlike systems could give access to the nuclear-size effect, which in turn can be used as an additional input for other determinations of fundamental constants.
- Finally, provided the mass m_{ion} of a high-Z ion is known to sufficient accuracy and m_{e} is taken from g factor measurements at lower nuclear charge number, the high-Z experimental result for g may be compared to a theoretical prediction, yielding a test of quantum electrodynamics for a bound particle subject to an external magnetic field and a strong Coulomb field.[1] Alternatively, one may invert the relation $g = g(\alpha)$ to solve for the fine-structure constant (important precondition: knowledge of nuclear size effect) [8,11]. The feasibility of the latter endeavour in various ranges of nuclear charge numbers will be discussed in the current article.

These examples illustrate the rich physics implied by g factor measurements in combination with the determination of atomic masses via Penning traps. Indeed, the g factor is a tremendous source of information regarding fundamental constants, fundamental interactions and nuclear properties.

This paper is organized as follows. In Section 2, we briefly discuss the importance and the status of atomic mass measurements for further advances. In Section 3, we describe a few details of two recent investigations [5,6] regarding one- and two-loop binding corrections to the g factor, and in Section 4, we discuss the asymptotics of the corrections for high quantum numbers, with a partially surprising result, before dwelling on connections of the g factor to nuclear effects and the fine-structure constant in Section 5. Conclusions are drawn in Section 6. An Appendix A is devoted to the current status of the free-electron anomaly.

2. Atomic mass measurements—present and future

A review of the current status of atomic mass measurements can be in found in ref. [12]. Experimental details regarding modern atomic mass measurements, with a special emphasis on hydrogenlike ions, can be found in refs. [13,14]. Regarding the current status of mass measurements, one may point out that some of the masses of S, Kr and Xe ions have recently been determined with an accuracy of better than 1 part in 10^{10} (Ref. [15]). For molecular ions, the accuracy has recently been pushed below 10^{-11} [16].

Recent measurements for the hydrogenlike ions $^{24}\mathrm{Mg}^{11+}$ and $^{26}\mathrm{Mg}^{11+}$ (Ref. [13]) and $^{40}\mathrm{Ca}^{19+}$ (Ref. [17]),[2] as well as for the lithiumlike ion $^{40}\mathrm{Ca}^{17+}$ (Ref. [17])[2] have reached an accuracy of about 5×10^{-10}. These experiments pave the way for accurate determinations of fundamental constants using g factor measurements in these systems. At the University of Mainz[3] (MATS collaboration) and at the University of Stockholm [17][2] (SMILE-TRAP), there are plans to significantly extend and enhance atomic mass measurements (including many more isotopes and nuclei) over the next few years, with accuracies below 1 part in 10^{11} or even 10^{12}. Eventually, one may even hope to determine the nuclear size effect of a specific ion by “weighing” the Lamb shift of the ground state. In the same context, one may point out that the masses of different charge states of ions are determined vice versa by adding and subtracting binding energies. This implies, e.g., that the mass of $^{12}\mathrm{C}^{5+}$ in terms of the mass of neutral carbon, $m(^{12}\mathrm{C}) = 12\,\mathrm{U}$, is given by

$$m(^{12}\mathrm{C}^{5+}) = m(^{12}\mathrm{C}) - 5\,m_{\mathrm{e}} + c^{-2}E_{\mathrm{B}}, \tag{2}$$

where $E_{\mathrm{B}} = 579.835(1) \times 10^{-9}\,\mathrm{U}$, c^2 is the cumulative binding energy for all five electrons [18]. This relation has proven useful in the determination of the electron mass [7].

In order to make a comparison to the accuracy of the free-electron determination of α, it is perhaps useful to remember that in the seminal work [19], the free-electron and positron anomaly has been determined to an accuracy 4×10^{-9}. This translates into a level of accuracy of about 4×10^{-12} for the g factor itself. The accuracy of the current value of α is 4×10^{-9} [3].

3. Calculation of the bound-electron *g* factor

The bound-electron g factor measures the energy change of a bound electron (hydrogenlike ion, spinless nucleus) under a quantal change in the projection of the total angular momentum with respect to an axis defined by a (weak) external magnetic field. In this sense, the g factor of a bound electron should rather be termed the g_J factor (according to the Landé formulation).

However, for S states, the total angular momentum number is equal to the spin quantum number, and therefore it has been common terminology not to distinguish the notation for g and g_J.

For a general hydrogenic state, the Dirac-theory g factor, denoted g_{D}, reads (see [9] and references therein)

$$g_{\mathrm{D}} = \frac{\kappa}{j(j+1)}\left(\kappa\frac{E_{n\kappa}}{m_{\mathrm{e}}} - \frac{1}{2}\right). \tag{3}$$

Here, E_{nj} is the Dirac energy, and the quantum numbers n, j and κ have their usual meaning. Throughout this article, we use natural units with $\hbar = c = \epsilon_0 = 1$.

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[1] See, e.g., Section 2.2 of P.D. Fainstein, et al., Stored Particle Atomic Research Collaboration (SPARC), Letter of Intent for Atomic Physics Experiments and Installations at the International FAIR Facility, 2004, unpublished.

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