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Generalized squeezed states

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A B S T R A C T

Squeezed states are one of the most useful quantum optical models having various applications in different areas, especially in quantum information processing. Generalized squeezed states are even more interesting since, sometimes, they provide additional degrees of freedom in the system. However, they are very difficult to construct and, therefore, people explore such states for individual setting and, thus, a generic analytical expression for generalized squeezed states is yet inadequate in the literature. In this article, we propose a method for the generalization of such states, which can be utilized to construct the squeezed states for any kind of quantum models. Our protocol works accurately for the case of the trigonometric Rosen–Morse potential, which we have considered as an example. Presumably, the scheme should also work for any other quantum mechanical model. In order to verify our results, we have studied the nonclassicality of the given system using several standard mechanisms. Among them, the Wigner function turns out to be the most challenging from the computational point of view. We, thus, also explore a generalization of the Wigner function and indicate how to compute it for a general system like the trigonometric Rosen–Morse potential with a reduced computation time.

1. Introduction

Squeezed states are interesting quantum optical systems exhibiting nonclassical properties [1–3]. They produce less noise in optical communication than a vacuum state. This is why squeezed light has various applications in different areas including optical communications [4], optical measurement [5], detection of gravitational waves [6,7], universal quantum computing [8], dense coding [9], etc. Squeezed states are also utilized in quantum metrology not only to improve the quantum metrology technique itself [10–12], but also to increase the sensitivity of gravitational wave detectors [13–15], especially the LIGO [16]. Moreover, squeezed light serves as a primary resource in continuous variable quantum information processing and, is utilized to distribute secret keys in quantum cryptography [17]. For an extensive list of applications one may refer, for instance [18,19].

The primitive idea of squeezing follows from the Heisenberg uncertainty principle. It is well-known that the coherent states of light minimize the uncertainty relation $\Delta x \Delta p = 1/2$, with both of the quadrature uncertainties being identical to each other $\Delta x = \Delta p = \sqrt{1/2}$ ($\hbar = 1$). Consequently, the uncertainty region of a coherent state can be represented by a circle in the optical phase space. However, for some states the uncertainty circle may be squeezed in one quadrature and elongated correspondingly in the other so that the uncertainty circle is deformed to form an ellipse and, the corresponding states are often familiar as squeezed states. Although, the uncertainty relation does not necessarily have to be minimized in the latter case, however, there are rare examples for squeezed states with minimum uncertainties [20,21], which are popular as *ideal squeezed states*. Nevertheless, so far we have discussed a particular type of squeezed states, namely the *quadrature squeezed* states, which are defined as the states whose standard deviation in one quadrature is less than that of the coherent states or a vacuum state. Squeezing can also occur in photon number distribution, and a state is said to be *number squeezed* if the photon number uncertainty of the corresponding states becomes lower than that of the coherent states. However, physically both of the scenarios refer to the notion of nonclassicality.

Coherent states are not nonclassical, in fact, they are the most classical analogue of quantum systems. But, the statement is true only for the coherent states of the harmonic oscillator, which are sometimes referred to the Glauber coherent states. However, there are various generalized coherent states [22–29] for which the quadrature and/or number squeezing occur and, thus, they are

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nonclassical. People sometimes refer these types of coherent states or any other nonclassical states to be *squeezed states* for which the quadrature and/or number squeezing occurs. It should be noted that in this article we do not refer these types of nonclassical states to be squeezed states, rather, we talk about a particular class of states as defined in Sec. 2, which are constructed in such a way that the quadrature squeezing is inherited to them by construction and, thus, they are always nonclassical.

Generalization of different quantum optical systems provides a deeper understanding, since sometimes it brings additional degrees of freedom in the system so that they can be applied more efficiently to physical models [29]. The essence of the generalization of several nonclassical states; such as, cat states [30–33], photon-added coherent states [34–37], pair-coherent states [38], binomial states [39], etc., have been explored in various contexts. Squeezed states are probably the most well-behaved nonclassical states which can be prepared more systematically and elegantly in the laboratory [40]. However, as per our knowledge, there is no particular form of the generalized squeezed states available in the literature, because they are extremely difficult to construct. In this article, we propose a generalization to such states followed by an example of a general system, viz., the trigonometric Rosen–Morse potential, where we apply our proposal directly. The reason behind choosing the Rosen–Morse potential for our analysis is mainly because it is a widely used model in optics, but mostly because the squeezed states for such model have not been studied notably. Therefore, we have the opportunity to explore in a two-fold way. Firstly, the generalization of the squeezed states is verified via an example through a popular model. Secondly, at the same time, we can shed light on the behavior of the Rosen–Morse squeezed states, which is inadequate in the literature. It should be noted that there are many articles available, for instance [41–53], entitled by "generalized squeezed states", however, they contain either the "generalized coherent states" having the squeezing properties or, they are any other type of nonclassical states whose quadrature and/or photon number is/are squeezed. However, according to our knowledge there is no trace of the generalization of the particular state that we discuss in the following section.

In Sec. 2, we discuss the detailed procedure for the generalization of the squeezed states by introducing a set of generalized ladder operators followed by an explicit analytic solution of the generalized squeezed states. In Sec. 3, we apply the obtained general solution to a particular type of model, namely the Rosen–Morse potential. Sec. 4 is composed of the analysis of nonclassicality and squeezing properties of the given example by means of the analysis of quadrature squeezing, sub-Poissonian photon statistics and Wigner distribution function. Finally, our concluding remarks are stated in Sec. 5.

2. Generalization

Squeezed states for harmonic oscillator $|\alpha,\delta\rangle_{\text{ho}}$ are constructed by operating the displacement operator $D(\alpha)=exp(\alpha a^\dagger-\alpha^* a)$ on the squeezed vacuum $S(\delta)|0\rangle$ [46]

$$|\alpha,\delta\rangle_{\text{ho}}=D(\alpha)S(\delta)|0\rangle,\quad S(\delta)=e^{\frac{1}{2}(\delta a^\dagger a^\dagger-\delta^* aa)}, \tag{1}$$

with $\alpha,\delta\in\mathbb{C}$ being displacement and squeezing parameters, respectively. Alternatively they can be formulated by performing the Holstein–Primakoff/Bogoliubov transformation on $S(\delta)$ arising from the solution of the following eigenvalue equation [50,54,55]

$$(a+\xi a^\dagger)|\alpha,\xi\rangle_{\text{ho}}=\alpha|\alpha,\xi\rangle_{\text{ho}},\ \xi=\frac{\delta}{|\delta|}\tanh(|\delta|),\ |\xi|<1, \tag{2}$$

which reduces to the coherent states for $\xi=0$. Generalization are usually carried out [50,56] by replacing the usual ladder operators $a,a^\dagger$ by the generalized ladder operators $A,A^\dagger$ in (2)

$$A^\dagger|n\rangle=\sqrt{k(n+1)}|n+1\rangle,\quad A|n\rangle=\sqrt{k(n)}|n-1\rangle, \tag{3}$$

with $k(n)$ being an operator-valued function of the number operator $n=a^\dagger a$ associated with generalized models. The generalized ladder operators (3) have been introduced long time back [26,28,31,32] in order to generalize various quantum optical models, especially coherent and cat states. These generalizations are mostly familiar as nonlinear generalization and they are well-established and widely accepted in the community. The existence of such states have also been found in many experiments using Kerr type nonlinearity and nonlinear cavity [57–59]. For further information in this regard one may follow some review articles in the context [29,60]. Note that, the generalized ladder operators (3) are given in such a form that $A^\dagger A$ behaves as the number operator of the generalized system and, therefore, the function $k(n)$ can be associated with the eigenvalues e_n of the model as follows

$$A^\dagger A|n\rangle=k(n)|n\rangle,\qquad k(n)\sim e_n, \tag{4}$$

which holds in general for the function $k(n)$. The appearance of additional constant terms in the eigenvalues can be realized by rescaling the composite system of A and $A^\dagger$ correspondingly. Therefore, by computing the eigenvalues of the system, one can construct the function $k(n)$ and, thus, various quantum optical states by using the nonlinear generalization procedure that is discussed above. Nevertheless, in order to solve the eigenvalue equation (2) in the generalized scenario, let us first expand the squeezed states in Fock basis

$$|\alpha,\xi\rangle=\frac{1}{\mathcal{N}(\alpha,\xi)}\sum_{n=0}^{\infty}\frac{\mathcal{J}(\alpha,\xi,n)}{\sqrt{k(n)!}}|n\rangle\ , \tag{5}$$

where $k(n)!=\prod_{i=1}^{n}k(i)$ and $k(0)=1$. By inserting (5) into (2) replaced by the generalized ladder operators (3), we end up with a three term recurrence relation

$$\mathcal{J}(\alpha,\xi,n+1)=\alpha\ \mathcal{J}(\alpha,\xi,n)-\xi\ k(n)\ \mathcal{J}(\alpha,\xi,n-1), \tag{6}$$

with $\mathcal{J}(\alpha,\xi,0)=1$ and $\mathcal{J}(\alpha,\xi,1)=\alpha$, which when solved one obtains the explicit form of the squeezed states [56]. But, the recurrence relation (6) is extremely difficult to solve for general $k(n)$, in fact, there are only few examples where it has been possible, that too for some particular models [56,61]. Notice that, Eq. (6) reduces to a simple form for $\xi=0$, which is the case of coherent states and, indeed, the corresponding solution leads to the nonlinear coherent states [28,31,62]

$$|\alpha\rangle=\frac{1}{\mathcal{N}(\alpha)}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{k(n)!}}|n\rangle\ , \tag{7}$$

which are the generalized version of the Glauber coherent states. Furthermore, in order to obtain the standard form of the harmonic oscillator squeezed states, we must consider $k(n)=n$ and, with this the recurrence relation (6) is solved to obtain

$$|\alpha,\xi\rangle_{\text{ho}}=\frac{1}{\mathcal{N}(\alpha,\xi)}\sum_{n=0}^{\infty}\frac{1}{\sqrt{n!}}\left(\frac{\xi}{2}\right)^{n/2}\mathcal{H}_n(\frac{\alpha}{\sqrt{2\xi}})|n\rangle\ , \tag{8}$$

where $\mathcal{H}_n(\alpha)$ denote the complex Hermite polynomials. We intend to provide a general solution applicable for any model, which is obtained by the general solution of (6) as follows

$$\mathcal{J}(\alpha,\xi,n)=\sum_{m=0}^{[n/2]}(-\xi)^m\alpha^{n-2m}g(n,m), \tag{9}$$