Doctopic: Nonlinear science ARTICLE IN PRESS PLA:25315

Physics Letters A ●●● (●●●●) ●●●–●●●

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Analysis of drivers' characteristics on continuum model with traffic jerk effect

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ARTICLE INFO

Article history:
Received 7 May 2018
Received in revised form 29 August 2018
Accepted 23 September 2018
Available online xxxx
Communicated by C.R. Doering

Keywords:
Continuum model
Traffic jerk
Driver's characteristics
Stability analysis

ABSTRACT

In this paper we propose an enhanced continuum model for traffic flow considering the effect of driver characteristics and traffic jerk. Based on the linear stability condition, the sufficient conditions of model stability is given. In the nonlinear stability analysis, we derive the KdV–Burger equation to describe the propagation characteristics of traffic density waves near the neutral stability curve. The simulation example verified that the driver characteristics and traffic jerk have a significant impact on the stability of the traffic flow and emissions.

1. Introduction

The rapid increase of car ownership has induced a series of traffic problems such as traffic congestion, traffic pollution and traffic safety issues. It is of great importance to understand the traffic flow propagation mechanism in order to effectively alleviate the traffic congestion and further put forwards the control strategies. To this end, a number of scholars have developed traffic flow models from different view of points [1–4]. In general, traffic flow models can be classified into microscopic and macroscopic models. The former includes car-following model [5–9] and the cellular automata model [10–14], while the latter includes lattice models [15–23] and gas kinetic models [24–26]. This paper is concerned with the macroscopic traffic flow models, more specifically the continuum models.

The optimal velocity car-following model [27], also known as the OV model, is most widely used microscopic car-following model. This seminal work was first proposed by Bando, in which the optimal speed was only determined by the headway with the forward vehicle. Due to the simplicity of the model, a number of extended works have been presented since the introduction of Bando's model. Generally, the incorporated new factors can be divided into the internal factors such as memory time [28–30], delay time [31–33] and forecast time [34–36], and the exogenous factors such as multiple ahead (or following) vehicle information [37–39], intelligent transportation system [40–42] and honk effect [43,44], and others [45–47]. Payne [48] proposed a continuum model to describe the traffic flow based on the work of Lighthill and Whitham [49,50]. However, as pointed out by Daganzo [51], the characteristic speeds could be greater than the macroscopic flow speed which exists a gas-like behaviour. Zhang [52] presented an improved continuum model which prevents vehicles from driving backward. Later, Jiang et al. [53] proposed a new anisotropic macroscopic continuum models based on the full velocity difference model. The proposed model allows the vehicles' speed to deviate from speed–density relationship. Since the low-order model facilitates analyzing the stop-and-go and the small disturbance instability traffic phenomena, such model has been gained wide attentions and extended by considering different characteristic, such as memory time [54–56], delay time [57] and forecast time [58–61], and the exogenous factors such as intelligent transportation system [62], lateral separation effect [63,64], bay-bus-stop effect [65] and driver's bounded rationality effect [66], and other realistic factors [67–71].

In the real operation, the traffic environment along the road would be rather complex. For instance, the pedestrians' non-compliance with the rules of traverse streets could result in the drivers' non-compliance with traffic rules and the chaotic phenomena on road, such as the present of sudden acceleration and deceleration [71]. The temporal dynamics of the acceleration and deceleration of a vehicle can

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https://doi.org/10.1016/j.physleta.2018.09.029

be represented by the jerk profile. Due to the existing of traffic jerk, a few attentions has been paid to its implications on traffic flow. For example, Liu et al. [72] firstly introduced traffic jerk into the continuum model, which verified that the traffic jerk contributes to the traffic flow stability. Later, Cheng et al. [73] presented a new continuum model based on full velocity difference on the basis of Liu et al. [72].

The underlying assumption of existing studies on traffic jerk [73–75], however, is that drivers is homogeneous, as such they made the same reaction under the same stimulus, which is unrealistic in practice. As concluded in the work by Peng et al. [76], according to microscopic characteristics such as age, gender and driving habits, drivers could be divided into two categories, namely aggressive and timid. Their results showed that the driving habits are differential between these two types of drivers, and that drivers of the same type could even perform differently due to various levels of driving experience. To the best of our knowledge, there is no literature on continuum model considering both the effect of drivers' characteristic and traffic jerk. Traffic jerk is commonly observed in traffic flow, while different types of drivers would adjust their driving speed in response to the traffic information in a different fashion. To increase the behavioural realism, we set out in the paper to fill this gap by incorporating these two factors jointly into the continuum model.

Based on the above discussion, the paper is organized as follows: in the second section, a new continuum model considering traffic jerk and drivers' characteristics is given. In the third part, the neutral stability curve and the KdV–Burger equation are obtained in linear and nonlinear analysis section, which will be verified by the simulation example. Finally, we conclude the main findings.

2. The continuum model and linear analysis

There exists high uncertainty for many traffic situations in the external environment, such as sudden braking and acceleration of vehicles. Based on the optimal velocity traffic flow model, Cheng et al. [73] proposed a car-following model considering such effect called "traffic jerk" effect, and the expression is given as follows:

$$\frac{dv_n(t)}{dt} = a\big[V\big(\Delta x_n(t)\big) - v_n(t)\big] + \eta\Delta v_n(t) - \lambda J_n(t) \tag{1}$$

where $v_n(t)$ is the instantaneous speed of the n-th vehicle at time t, and $\Delta x_n(t)$ is the instantaneous headway of the n-th vehicle at time t, $\Delta v_n(t)$ is the velocity difference between the n-th vehicle and the $n-1$-th vehicle at time t, η is the weight coefficient of $\Delta v_n(t)$, $J_n(t) = \frac{dv_n(t)}{dt} - \frac{dv_n(t-1)}{dt}$ represents the traffic jerk of n-th vehicle at time t, which is defined as the acceleration difference between the current and the preceding moment. λ is the jerk parameter, $0 \le \lambda \le \frac{2}{3}$. a is the sensitivity of drivers.

In a real situation, drivers present heterogeneity in that they may react differently to the traffic information. Generally, the drivers are divided into two categories, namely, aggressive and timid [76–78]. The former group tends to speed up with the forecasted future traffic condition, whereas the latter group would be more prudent in acceleration. Specifically, timid drivers often sense the traffic information at time t and adjust their driving speed at a later time $t+\sigma_1\tau$. In other words, there exists delay time before vehicle acceleration for those timid ones. Compared to the timed drivers, aggressive drivers have more capability in forecasting the future state, such that they are likely to adjust their driving speed to the perceived optimal speed at time $t-\sigma_2\tau$ in advance. To this end, we introduce two new set of variables $V(\Delta x_n(t+\sigma_1\tau))$ and $V(\Delta x_n(t-\sigma_2\tau))$ to reflect the driving characteristics of timid and the aggressive drivers. Where $\sigma_1\tau$ and $\sigma_2\tau$ represent the anticipation time for aggressive drivers and the delay time for the timid drivers, respectively. σ_1 and σ_2 denote the anticipation coefficients for aggressive drivers and timid drivers, respectively. τ is the time scale.

As a result, a new car-following models considering the drivers characteristics and traffic jerk effect is proposed as follows:

$$\frac{dv_n(t)}{dt} = a\big[pV\big(\Delta x_n(t+\alpha_1\tau)\big) + (1-p)V\big(\Delta x_n(t-\alpha_2\tau)\big) - v_n(t)\big] + \eta\Delta v_n(t) - \lambda J_n(t) \tag{2}$$

where α_1, α_2 indicates driver's forecast and delay time weight coefficient respectively. For simplicity and analytical tractability, we assume that $\alpha_1 = \alpha_2 = \alpha$. The parameter $0 \le p \le 1$ indicates the intensity effect of driver characteristic. When $0.5 < p \le 1$, the drivers perform aggressively; when $0 \le p < 0.5$, the driver's characteristics are dominated by timid; when $p = 1$ ($p = 0$), then the drivers are completely aggressive (timid).

In order to derive the continuum model, we first carry out the Taylor expansion of $\Delta x_n(t+\alpha_1\tau)$ and $\Delta x_n(t-\alpha_2\tau)$ and then ignore the higher order terms, thereby we obtain the following equations:

$$\Delta x_n(t+\alpha\tau) = \Delta x_n(t) + \alpha\tau\Delta v_n(t) \tag{3}$$

$$\Delta x_n(t-\alpha\tau) = \Delta x_n(t) - \alpha\tau\Delta v_n(t) \tag{4}$$

According to Eqs. (3) and (4), we have

$$V\big(\Delta x_n(t+\alpha\tau)\big) = V\big(\Delta x_n(t) + \alpha\tau\Delta v_n(t)\big) = V\big(\Delta x_n(t)\big) + \alpha\tau\Delta v_n(t)V'\big(\Delta x_n(t)\big) \tag{5}$$

$$V\big(\Delta x_n(t-\alpha\tau)\big) = V\big(\Delta x_n(t) - \alpha\tau\Delta v_n(t)\big) = V\big(\Delta x_n(t)\big) - \alpha\tau\Delta v_n(t)V'\big(\Delta x_n(t)\big) \tag{6}$$

Therefore, Eq. (2) can be rewritten as

$$\frac{dv_n(t)}{dt} = a\big[V\big(\Delta x_n(t)\big) - v_n(t)\big] + \big[\eta - \alpha(2p-1)\rho^2 V_e'(\rho)\big]\Delta v_n(t) - \lambda J_n(t) \tag{7}$$

In order to transform the microscopic model (7) into its respective macroscopic version, the following method can be used to convert the micro variables to macro variables, where

$$v_n(t) \to v(x,t), \qquad v_{n+1}(t) \to v(x+\Delta,t) \tag{8}$$

$$V\left(\frac{1}{\rho}\right) \to V_e(\rho), \qquad V'\left(\frac{1}{\rho}\right) \to -\rho^2 V_e'(\rho) \tag{9}$$

Please cite this article in press as: C. Zhai, W. Wu, Analysis of drivers' characteristics on continuum model with traffic jerk effect, Phys. Lett. A (2018), https://doi.org/10.1016/j.physleta.2018.09.029