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Riemann-Finsler Geometry and Lorentz-Violating Scalar Fields

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Abstract

The correspondence between Riemann-Finsler geometries and effective field theories with spin-independent Lorentz violation is explored. We obtain the general quadratic action for effective scalar field theories in any spacetime dimension with Lorentz-violating operators of arbitrary mass dimension. Classical relativistic point-particle lagrangians are derived that reproduce the momentum-velocity and dispersion relations of quantum wave packets. The correspondence to Finsler structures is established, and some properties of the resulting Riemann-Finsler spaces are investigated. The results provide support for open conjectures about Riemann-Finsler geometries associated with Lorentz-violating field theories.

1. Introduction

A correspondence between a large class of Riemann-Finsler geometries [1, 2] and realistic effective field theories with explicit Lorentz violation has recently been identified [3]. The underlying idea is that the classical trajectory of a relativistic wave packet in the presence of perturbative Lorentz violation can be mapped via a suitable continuation to a geodesic in a Riemann-Finsler space. The correspondence is of both mathematical and physical interest. On the mathematics side, it provides a rich source of examples of Riemann-Finsler geometries that are perturbatively close to Riemann geometry. One example uncovered in this way is a calculable geometry, called b space, that is a natural complement of Randers geometry [4]. The known classification and enumeration of Lorentz-violating effects may also permit a parallel classification of the corresponding Riemann-Finsler spaces. On the physics side, the correspondence is expected to shed light on the poorly understood geometric structure of theories of gravitation with explicit Lorentz breaking [5]. Also, in analogy with the geometric interpretation of Zermelo navigation [6] in terms of Randers geometry [7], the correspondence can be applied to geometric descriptions of physical systems [8]. Related concepts are explored in various contexts in a broad recent literature [9–24].

In nature, Lorentz and CPT violation could arise from an underlying theory combining gravity with quantum physics such as strings [25]. Observable effects on the behavior of known fundamental particles can be inferred from the comprehensive realistic effective field theory for Lorentz violation incorporating the Standard Model of particle physics and General Relativity, called the Standard-Model Extension (SME) [5, 26]. Most of the known fundamental particles have spin, with only the Higgs boson being a spinless field in the Standard Model. A nonzero spin complicates the particle trajectory in part because it involves intrinsically quartic dispersion relations rather than intrinsically quadratic ones [27]. However, even for a particle with nonzero spin, a subset of Lorentz-violating effects are spin independent and hence can be handled as though the particle had zero spin. The combination of relevance and comparative simplicity enhances interest in the correspondence between Riemann-Finsler geometries and the trajectories of particles experiencing spin-independent Lorentz violation.

In this work, we construct the general effective scalar field theories in any spacetime dimension that contain explict perturbative spin-independent Lorentz-violating operators of arbitrary mass dimension. The results are used to obtain the general classical lagrangian describing the propagation of a relativistic spinless point particle in the presence of Lorentz violation. The correspondence between the classical lagrangian and Riemann-Finsler geometries is established, and some properties of the latter are studied. Among the results is a set of calculable y-global Riemann-Finsler geometries that are perturbatively close to Riemann geometries. The properties of these spaces offer support for some unresolved conjectures about Riemann-Finsler geometries associated with Lorentz-violating field theories.

2. Scalar field theory

Consider a complex scalar field $\phi(x^\mu)$ of mass m in n-dimensional spacetime with Minkowski metric $\eta_{\mu\nu}$ of negative signature for $n > 2$. The effective quadratic Lagrange density describing the propagation of ϕ in the presence of arbitrary Lorentz-violating effects can be written in the form

$$\mathcal{L}(\phi, \phi^\dagger) = \partial^\mu \phi^\dagger \partial_\mu \phi - m^2 \phi^\dagger \phi - \tfrac{1}{2}\left(i\phi^\dagger (\hat{k}_a)^\mu \partial_\mu \phi + \text{h.c.}\right) + \partial_\mu \phi^\dagger (\hat{k}_c)^{\mu\nu} \partial_\nu \phi, \quad (1)$$

where $(\hat{k}_a)^\mu$ and $(\hat{k}_c)^{\mu\nu}$ are operators constructed as series of even powers of the partial spacetime derivatives ∂_α. Since Lorentz violation is expected to be small in nature and perhaps