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Chemometrical approach to the determination of the fractal dimension(s) of real objects

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Abstract

Fractal dimension has been recognized for a long time as a useful parameter for shape characterisation. However, the use of the fractals concept requires the visual inspection of the Richardson plot which hampers the practical applications. In this paper a fully automatical method for the analysis of the Richardson plot is described. A clustering approach is adopted instead of the visual inspection of Richardson plot. Cases which explain the application of the method are examined.

Keywords: Fractal dimensions; Shape characterization

1. Introduction

The concept of fractals was originally introduced in one of the earlier papers by Mandelbrot [1] in which he tried to resolve the paradox encountered by Richardson in his attempt to answer the question 'how long is the coastline of Great Britain'. The answer to this question essentially seems to depend on the yardstick used to measure the length of the coastline. Richardson observed that if the measured coastline was plotted against the size of the yardstick, λ, using logarithmic axes (Richardson plot), the result was a straight line. Furthermore, the slope of the line was different for different boundaries. More irregular coastlines as judged by human observers always result in a line with a higher slope. The slope is negative, since the largest value for the coastline length is obtained with the shortest yardstick. The magnitude of the slope of the line is between zero and one. The fractal dimension of the coastline is just this magnitude plus 1, the topological dimension of a line.

This concept was found to be useful to determine the fractal dimension of not only the coastlines, but also of different natural objects: aerosol particles, biological cells, etc. It was shown (see, e.g. [2]) that for a variety of objects at various magnifications the data did not follow the ideal straight line (single fractal) expected for a (true) fractal. Practically in all cases the data on a Richardson plot can be satisfactorily explained as two straight line segments having different slopes (bifractals or, in general, multifractals). Kaye's description of the two line segments of the Richardson plots uses the terms 'structural' and 'textural' for the two different regions. At fine scales a 'textural' dimension is given by the slope of the line at small values of λ, while at large scale the 'structural' characteristics of the object emerge.

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When a Richardson plot has been constructed (various techniques can be found in [2,3]) it is necessary to fit a straight line to the data to determine the slope (or slopes in the case of a multifractal) and hence the fractal dimension(s). The main difficulty is that, as it has been pointed by Mandelbrot, in the real world any fractal description of a natural boundary would have inspection limits. It means that fractality of a real object can be observed only within some limited intervals of the yardstick sizes. However, it is not always evident where the best straight line segment(s) on the graph occur. Moreover some multifractal objects may show continuous gradients of fractal change [4] and it is not easy to determine where the best breakpoint between two straight lines of the graph is. There are no criteria proposed up to date for preferable breakpoint selection in such cases. Up to our knowledge the straight line segment of a Richardson plot is mainly detected by visual inspection [4–7]. Obviously, this visual line fitting can lead to problems. Here we propose a method which eliminates these difficulties and reveals the correct fractal dimension(s) without visual inspection of a Richardson plot. This procedure allows to fully automate the determination of the fractal dimension. Such automation is important in, e.g., electron microscopic investigation of microscopic particles. In environmental research, the composition and shape of hundreds or thousands of atmospheric aerosol particles are determined. Their fractal dimension, together with their chemical composition is used to identify the particles and to assign them to possible sources [7].

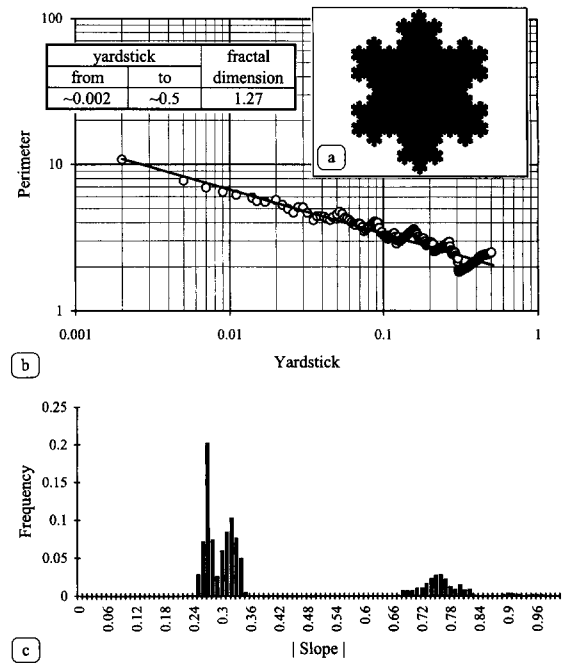

Fig. 1. (a) The triadic Koch island after 5 iterations with theoretical fractal dimension log 4/log 3 = 1.2618...; (b) corresponding Richardson plot (here and in the next figures yardstick and perimeter are shown in fractions of the maximum Feret diameter); (c) the frequency histogram, obtained from the Richardson plot shown ($k = 4$, $\sigma = 0.01$). Its maximum corresponds to a fractal dimension of 1.27.

2. Theory

The fractal dimension(s) of an object is (are) determined from its digital image (e.g. as obtained with a scanning electron microscope). The object is discriminated from the background by converting the original grey level image into a binary image. Next the contour of the object is obtained using a classical boundary following technique [7], resulting in a set of contour points $\{x_i, y_i\}$.

2.1. Construction of the Richardson plot

Construction of the Richardson plot involves the determination of the perimeter of the object for various lengths of the yardstick λ. In the Richardson plot the logarithm of the perimeter is plotted against the logarithm of the yardstick length (Fig. 1b).

For a given yardstick size λ, the perimeter of the object is determined as follows. Starting at some arbitrary contour point (x_s, y_s) the next point on the contour (x_n, y_n) in clockwise direction is located which has distance $d_j = \sqrt{(x_s - x_n)^2 + (y_s - y_n)^2}$ as close as possible to λ. This point is then used to locate the next point on the contour that satisfies this condition. The process is repeated until the distance between the last located point and the starting point is less than λ. The perimeter is the sum of all distances d_j including the distance between the last located point and the starting point. This method is referred to as the 'hybrid' method and is discussed in detail in [8].

The length of the yardstick λ usually varies between 0.001 and 0.5 times the maximum Feret diam-