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The randomization model for experiments in block designs and the recovery of inter-block information

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Abstract

A general randomization model for experiments in block designs and conditions for obtaining the best linear unbiased estimators of treatment parametric functions under the model are re-examined. It is shown how the best linear unbiased estimators under the overall randomization model can be obtained when the variance components are known. Furthermore, a general method of estimating these, usually unknown, variance components is described and properties of the resulting estimators of treatment parametric functions are considered. In connection with this the recovery of inter-block information in a general block design is discussed.

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1. Introduction and preliminaries

One of the problems of interest related to experiments in block designs is the utilization of between block information for estimating treatment parametric contrasts. This problem is known in the literature, after Yates (1939, 1940), as the recovery of inter-block information. It is related to the common practice of randomizing the experimental material before it is subjected to experimental treatments. Therefore, methods dealing with the recovery of inter-block information have to be based on a properly derived randomization model. The purpose of the present paper is to reconsider such a model and to re-examine the principles underlying the recovery of

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inter-block information in a general block design. By that it is to extend the recent update of the problem given by Shah (1992), in which only equiblock-sized designs are considered.

The notation and terminology of the present paper follows essentially that used by Pearce (1983, Chapter 3) and recently by Caliński (1993). Thus, a block design for v treatments in b blocks is described by its $v \times b$ incidence matrix $\boldsymbol{N}$, from which $\boldsymbol{r} = \boldsymbol{N}\boldsymbol{1}_b$ and $\boldsymbol{k} = \boldsymbol{N}'\boldsymbol{1}_v$ are the column vectors of treatment replications and of block sizes, respectively, giving $n = \boldsymbol{1}_v'\boldsymbol{r} = \boldsymbol{1}_b'\boldsymbol{k}$ as the total number of units (plots) used in the experiment ($\boldsymbol{1}_b$ and $\boldsymbol{1}_v$ being column vectors of ones, of the indicated dimensions). The matrix $\boldsymbol{N}$ can be defined as $\boldsymbol{N} = \boldsymbol{\Delta}\boldsymbol{D}'$, where $\boldsymbol{\Delta}'$ is the $n \times v$ design matrix for treatments, and $\boldsymbol{D}'$ is the $n \times b$ design matrix for blocks. These matrices provide the diagonal matrices $\boldsymbol{r}^{\delta} = \boldsymbol{\Delta}\boldsymbol{\Delta}'$ and $\boldsymbol{k}^{\delta} = \boldsymbol{D}\boldsymbol{D}'$ that are used in some formulae, either directly or as $\boldsymbol{r}^{-\delta} = (\boldsymbol{r}^{\delta})^{-1}$ and $\boldsymbol{k}^{-\delta} = (\boldsymbol{k}^{\delta})^{-1}$, respectively. Furthermore, distinction is made between the potential, i.e. available, number of blocks, N_B, from which a choice can be made, and the number, b, of those actually chosen for the experiment. The usual situation is that $b = N_B$, but in general $b \leqslant N_B$. Similarly, it will be convenient to distinct between the potential (available) number of units within a block and the number of those actually used in the experiment.

2. The randomization model

2.1. The model and its properties

According to one of the basic principles of experimental design (cf. Fisher, 1925, Section 48), the units are to be randomized before they enter the experiment. Suppose that the randomization is performed by randomly permuting labels of the N_{B} available blocks and by randomly permuting labels of the available units in each given block, all the $1 + N_b$ permutations being carried out independently (cf. Nelder, 1954; White, 1975). Then, assuming the usual unit-treatment additivity (cf. Nelder, 1965b, p. 168; White, 1975, p. 560; Bailey, 1981, p. 215; Kala, 1991, p. 7), and also assuming, as usual, that the technical errors are uncorrelated, with zero expectation and a constant variance, and independent of the unit responses to treatments (cf. Neyman, 1935, pp. 110–114, 145; Kempthorne, 1952, p. 132, Section 8.4; Ogawa, 1963, p. 1559), the model of the variables observed on the n units actually used in the experiment can be written in matrix notation as

$$\boldsymbol{y} = \boldsymbol{\Delta}'\boldsymbol{\tau} + \boldsymbol{D}'\boldsymbol{\beta} + \boldsymbol{\eta} + \boldsymbol{e}, \tag{2.1}$$

where $\boldsymbol{y}$ is an $n \times 1$ vector of observed variables $\{y_{\ell(j)}(i)\}$, $\boldsymbol{\tau}$ is a $v \times 1$ vector of treatment parameters $\{\tau_i\}$, $\boldsymbol{\beta}$ is a $b \times 1$ vector of block random effects $\{\beta_j\}$, $\boldsymbol{\eta}$ is an $n \times 1$ vector of unit errors $\{n_{\ell(j)}\}$ and $\boldsymbol{e}$ is an $n \times 1$ vector of technical errors $\{e_{\ell(j)}\}$, i being the treatment label and $\ell(j)$ that of the assigned unit ℓ in block j ($i = 1, \ldots, v$; $\ell = 1, \ldots, k_j$; $j = 1, \ldots, b$).