Physica A 515 (2019) 707–714

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Partial component synchronization on chaotic networks

Fengbing Li [a,b], Zhongjun Ma [b,*], Qichang Duan [a]

[a] School of Automation, Chongqing University, Chongqing, 400044, China
[b] School of Mathematics and Computing Science, Guilin University of Electronic Technology, Guilin, Guangxi, 541004, China

HIGHLIGHTS

- Partial component synchronization is a kind of group dynamics behavior weaker than identical synchronization.
- In this paper, the definition of partial component synchronization is given, and the stability theory of partial variables is applied to study it.
- Several sufficient conditions for partial component synchronization to be realized on the network are derived.

ARTICLE INFO

Article history:
Received 2 August 2018
Received in revised form 21 September 2018
Available online xxxx

Keywords:
Synchronization
Chaos
Complex network
Partial variable stability

ABSTRACT

As for the dynamical networks which consist of some high-dimensional nonlinear systems, the problems that researchers are concerned with are usually the asymptotic convergence on some components (rather than all components) of node's state variables under certain condition. This means that partial component synchronization is more meaningful than identical synchronization in some cases. In this paper, the definition of partial component synchronization is given, and then the problem of partial component synchronization on a class of chaotic dynamical networks is investigated. By using matrix theory, stability theory and the hypothesis that several components in the solution vector of a single uncoupled node are ultimately dissipative, some sufficient conditions on partial component synchronization in the chaotic dynamical networks are derived. Finally, numerical simulations are shown to demonstrate the correctness of the theoretical results.

1. Introduction

Many facts show that each uncoupled node in a nonlinear dynamical network has its own dynamic behavior and that it evolves with time according to some dynamic laws under the corresponding initial conditions. When the couplings work, the properties of group dynamics of nodes in the network show a certain relationship, which is called synchronization in the term of dynamics. Since Pecora and Carroll [1,2] discovered the synchronization of chaotic systems in the 1990s, many literatures on chaos synchronization have appeared [3–11], such as identical (or complete) synchronization, cluster (or partial) synchronization, phase synchronization, outer synchronization and generalized synchronization. Identical synchronization (or cluster synchronization) above is the asymptotic homogeneity of all state components in all oscillators (or all oscillators within each cluster). These research results focus mainly on asymptotic convergence of all components in state variables.

In nonlinear dynamical network model, the dynamic equation of a single oscillator is usually a high-dimensional nonlinear system, and its dynamic behavior is very complex. Furthermore, in some cases, some components of state variables are even affected by unknown factors. Accordingly, the dynamic evolution of the network model is more complicated under

* Corresponding author.
E-mail address: mazhongjun@guet.edu.cn (Z. Ma).

https://doi.org/10.1016/j.physa.2018.10.008
0378-4371/

the coupling action. Therefore, it is often difficult, sometimes impossible, to study identical synchronization behaviors and its implementation conditions on the model. On the other hand, when considering the synchronization problem of a complex dynamical network model, which is composed of some high dimensional nonlinear systems by couplings with each other, the focus may be on the asymptotic convergence of some components (rather than all components) of node's state variables under certain condition. For example, when several aircraft fly side-by-side (or radial) in a flight performance, the component of displacement in the forward direction of flight is the same, while the components of displacement in the other direction are not the same. Essentially, this is a problem of partial component consistency of displacement. Inspired by the above depiction and the research results on partial component consensus [12], the problem of partial component synchronization on chaotic networks is considered in this paper. Compared with identical synchronization (all state components of all nodes are required to achieve asymptotic convergence), partial component synchronization is more general, and it is a kind of group dynamics behavior weaker than identical synchronization. As far as we know, there is no literature on partial component synchronization in network systems.

The main contents of this paper are as follows. In Section 2, we introduce the knowledge of partial variable stability and matrix theory. In Section 3, we give the concept of partial component synchronization. Then, under the condition that some components of the solution vector in a single uncoupled oscillator are eventually dissipative, the problem of partial component synchronization in chaotic dynamical networks is discussed. By using the matrix theory and stability theory, some sufficient conditions for partial component synchronization in the network system are derived. In Section 4, numerical simulation is carried out to verify correctness of the theoretical results. In Section 5, the discussion and summary are given.

2. Preliminaries

First of all, some research results about stability theory used in this paper are given [12,13]. Consider a system of nonautonomous ordinary differential equations

$$\frac{dx}{dt} = F(t, x), \tag{1}$$

where $F(t, x) \in C[R^+ \times R^n, R^n]$, $F(t, 0) \equiv 0$, $x = (y, z)^{\mathrm{T}} = (x_1, \ldots, x_m, x_{m+1}, \ldots, x_n)^{\mathrm{T}} \in R^n$, $y = (x_1, \ldots, x_m)^{\mathrm{T}} \in R^m$, $z = (x_{m+1}, \ldots, x_n)^{\mathrm{T}} \in R^p$, $(m + p = n)$, $\|x\| = (\sum_{i=1}^{n} x_i^2)^{1/2}$, $\|y\| = (\sum_{i=1}^{m} x_i^2)^{1/2}$, $\|z\| = (\sum_{i=m+1}^{n} x_i^2)^{1/2}$, $R^+ = [0, +\infty)$, $t \in R^+$.

Definition 1 ([13]). The trivial solution of the system (1) is said to be stable with respect to the variable y, if $\forall \varepsilon > 0$, $\forall t_0 \in R^+$, $\exists \delta(t_0, \varepsilon) > 0$, $\forall x_0 \in S_{\delta(n)} = \{x\|x\| < \delta\}$, such that

$$\|y(t, t_0, x_0)\| < \varepsilon (t \geq t_0).$$

Definition 2 ([13]). The trivial solution of the system (1) is said to be attractive with respect to the variable y, if $\forall t_0 \in R^+$, $\exists \sigma(t_0) > 0$, $\forall \varepsilon > 0$, $\forall x_0 \in S_{\delta(t_0)} = \{x\|x\| \leq \sigma(t_0)\}$, $\exists T(t_0, x_0, \varepsilon) > 0$, such that

$$\|y(t, t_0, x_0)\| < \varepsilon$$

for $t \geq t_0 + T$. Furthermore, $S_{\sigma(t_0)}$ called the region of attraction with respect to the variable y.

Definition 3 ([13]). The trivial solution of the system (1) is said to be asymptotically stable with respect to the variable y, if it is both stable and attractive with respect to the variable y.

Definition 4 ([13]). The function φ is said to belong to K class function, if $\varphi \in C[R^+, R^+]$ (or $C[(0, r), R^+]$) is continuous, strictly monotone increasing, and $\varphi(0) = 0$. It is recorded as $\phi \in K$.

Lemma 1 ([13]). *Let ϕ, ψ and α belong to K class functions. If there is a function $V(t, x)$ with*

$$\varphi(\|y\|) \leq V(t, x) \leq \psi(\|y\|),$$

such that its derivative

$$\left.\frac{dV}{dt}\right|_{(1)} \leq -\alpha(\|y\|),$$

then the trivial solution of the system (1) is asymptotically stable with respect to the variable y.

Remark 1. For the plane system $\mathrm{d}u/\mathrm{d}t = u$, $\mathrm{d}v/\mathrm{d}t = -v$, the trivial solution is asymptotically stable with respect to the variable v, but unstable.

Following, two lemmas about matrix theory are given.