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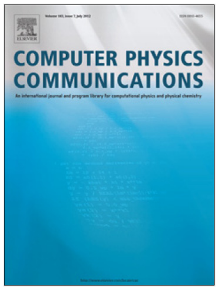

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Multiple-Relaxation-Time Lattice Boltzmann scheme for Fractional Advection-Diffusion Equation

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Abstract

Partial differential equations (p.d.e) equipped with spatial derivatives of fractional order capture anomalous transport behaviors observed in diverse fields of Science. A number of numerical methods approximate their solutions in dimension one. Focusing our effort on such p.d.e. in higher dimension with Dirichlet boundary conditions, we present an approximation based on Lattice Boltzmann Method with Bhatnagar-Gross-Krook (BGK) or Multiple-Relaxation-Time (MRT) collision operators. First, an equilibrium distribution function is defined for simulating space-fractional diffusion equations in dimensions 2 and 3. Then, we check the accuracy of the solutions by comparing with *i)* random walks derived from stable Lévy motion, and *ii)* exact solutions. Because of its additional freedom degrees, the MRT collision operator provides accurate approximations to space-fractional advection-diffusion equations, even in the cases which the BGK fails to represent because of anisotropic diffusion tensor or of flow rate destabilizing the BGK LBM scheme.

Keywords:
Fractional Advection-Diffusion Equation, Lattice Boltzmann method, Multiple-Relaxation-Time, Random Walk, Stable Process.

1. Introduction

Among diverse non-Fickian transport behaviors observed in all fields of Science, heavy tailed spatial concentration profiles recorded on chemical species, living cells or organisms, suggest displacements more rapid than the classical Advection Diffusion Equation (ADE) predicts [1–5]. Such super-dispersive phenomena include plumes that lack finite second moment, or whose mean and peak do not coincide (see [6]). Possible explanations may be large scale heterogeneity or multiple coupling between many simple sub-systems which separately would not exhibit such abnormalities. Similar strange behaviors are observed often enough to suggest exploring alternative models as fractional partial differential equations. It turns out that [6–10] many tracer tests in rivers and underground porous media are accurately represented by the more general conservation equation

$$\frac{\partial C}{\partial t}(\mathbf{x},t)+\boldsymbol{\nabla}\cdot(\mathbf{u}C)(\mathbf{x},t)=\boldsymbol{\nabla}\cdot\overline{\overline{\mathbf{D}}}(\mathbf{x})\boldsymbol{\mathcal{F}}^{\boldsymbol{\alpha}pg}(C)+S_c(\mathbf{x},t). \tag{1}$$

It models mass spreading for passive solute at concentration C in incompressible fluid flowing at average flow rate $\mathbf{u}=\sum_{\mu=1}^{d}u_\mu\mathbf{b}_\mu$ super-imposed to small scale velocity field whose complexity causes non-Fickian dispersive flux $-\overline{\overline{\mathbf{D}}}\boldsymbol{\mathcal{F}}^{\boldsymbol{\alpha}pg}(C)$. The space variable $\mathbf{x}$ belongs to some domain Ω of $\mathbb{R}^d$, and is described in the orthonormal basis $\{\mathbf{b}_\mu;\text{ for }\mu=1,\ldots,d\}$ of $\mathbb{R}^d$ by its coordinates noted x_μ: greek subscripts refer to spatial coordinates. Moreover S_c is a

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