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Estimation of confidence intervals for radial emissivity and optimization of data treatment techniques in Abel inversion

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Abstract

A novel method is described for finding confidence intervals for radially resolved emissivity derived from Abel inversion. The essence of the method is that the measured lateral emission profile (here a profile is defined as a line-of-sight spatial map) is split into two components, namely (a) an estimated noise-free lateral emission profile that mimics the lateral emission profile in the absence of measurement noise and (b) a noise profile that estimates the magnitude of noise along the lateral profile. Through random-number generation, noise with a magnitude defined by the noise profile is then artificially added to the estimated noise-free lateral profile. If this noise-addition and subsequent Abel inversion is iterated for a sufficient number of cycles, a statistical distribution of the Abel-inverted radial profiles can be obtained and confidence intervals can thus be estimated. It was verified that an effective means for the estimation of the noise-free lateral profile involves approximation of the lateral emission profile through polynomial fitting. A noise profile can then be obtained by assuming that the residual between the experimental lateral profile and the fitted profile is due solely to noise. The validity of this methodology was verified with seven lateral test profiles. In addition, the parameters that are commonly used in data treatment by Abel inversion are optimized. It was found that 100–300 data points are sufficient for accurate Abel inversion using the Nestor-Olsen inversion algorithm; more data points provide only minimal improvement in the inversion. Also, it was found that most commonly observed lateral emission profiles can be satisfactorily approximated by polynomials with a degree of fifteen. Further, polynomial fitting can be used to partially filter out noise in the lateral profile. Depending on the shape of the lateral profile and the magnitude of the noise, the optimal polynomial order, in general, ranges from ten to fifteen. For higher-degree polynomials, the effectiveness of a polynomial as a noise filter decreases and severely degrades the quality of the Abel-inverted radial profile.

Keywords: Abel inversion; Inductively coupled plasma; Microwave induced plasma; Plasma

1. Introduction

Emission from most sources (e.g., inductively coupled plasma [1–3], microwave induced plasma [4,5], spark and arc discharge [6,7], laser induced plasma [8], etc.) is usually measured laterally and therefore contains contributions from all the plasma radial positions along the line of measurement. Since in most cases the emission profile of the source is not radially uniform, techniques to extract radial information about the emission source have been developed. One such technique is Abel inversion, which allows determination of the radial emission pattern from a complete lateral emission profile if the plasma has cylindrical symmetry [9–16]. An approximate Abel inversion for asymmetric emission sources has also been developed [3,17–20]. However, in the present paper, only optically thin emission sources having cylindrical symmetry are discussed. For optically thin emission sources with cylindrical symmetry (Fig. 1), the laterally measured intensity, $I(x)$, is the one-dimensional projection of the two-dimensional, cylindrically symmetric function having $\varepsilon(r)$ as a radial slice [14]. The Abel transform relates the lateral emission, $I(x)$ (in energy per unit time, unit area perpendicular to the x-direction and unit solid angle—W cm^{-2} sr^{-1}), to the radial emission, $\varepsilon(r)$ (in energy per unit time, unit volume and unit solid angle—W cm^{-3} sr^{-1}), by the following integral equations

$$I(x) = 2\int_x^R \frac{r\varepsilon(r)}{\sqrt{r^2 - x^2}}\,dr \tag{1}$$

$$\varepsilon(r) = \frac{-1}{\pi}\int_r^R \frac{I'(x)}{\sqrt{x^2 - r^2}}\,dx \tag{2}$$

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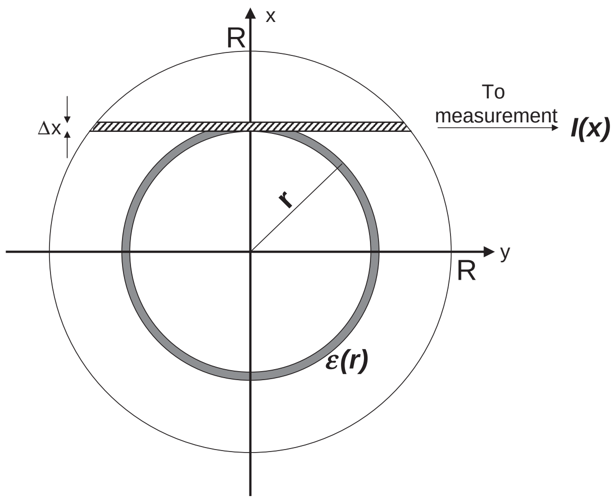

Fig. 1. Schematic diagram showing the relationship between radial and lateral emission profiles (cf. Eqs. (1) and (2) for definition of terms).

where x is the lateral displacement from the plasma center, r is the radial distance from the center of the source and R is the radius of the source.

It is well known that the Abel-inverted radial profile exhibits the greatest error in the center of the source (i.e., when r approaches zero), since measurement error and noise propagate from the edge to the center of the source (cf. Eq. (2)). Moreover, the Abel inversion is very sensitive to noise in the lateral profile because the derivative of $I(x)$ (i.e., $I'(x)$) is used (cf. Eq. (2)). An example is given in Fig. 2. Fig. 2a shows an assumed lateral emission profile with no noise, and with 0.1%, 0.3% and 1% added noise; Fig. 2b shows the resulting radial profiles obtained by direct Abel inversion without any data smoothing or noise filtering. For clarity, the lateral profiles in Fig. 2a are offset from each other by 0.1 unit. In this paper, the noise level is defined as the relative standard deviation (%) of the signal by assuming a Gaussian noise distribution (i.e., 1% noise means that ~68% of the noise magnitude is within 1% of the signal strength). It is evident from Fig 2a and b that even a noise level of 0.1% in the lateral emission profile significantly degrades the quality of the computed radial profile. As a result of this sensitivity to noise, methodologies to minimize variation in the measured lateral-emission profile have been the subject of numerous papers [21–24]. However, even these methodologies cannot completely eliminate noise, and a resulting radial profile that is noise-defect free cannot be guaranteed.

Considering this sensitivity to noise, there is a need to develop ways to distinguish whether the difference between two dissimilar Abel-inverted profiles is significant or merely due to random error. A common way to study the effect of noise in an Abel-inverted profile and to estimate the region of the profile that is relatively "noise-free" is by using an assumed $\varepsilon(r)$ profile that looks similar to the experimental one in two steps [3,5,9]. First, a lateral emission profile, $I(x)$, is computed from the assumed $\varepsilon(r)$ by applying Eq. (1). Second, various degrees of noise are added to this lateral profile and all the data manipulation procedures (e.g., smoothing, noise filtering, Abel inversion) identical to those applied to the measured data are subsequently performed; one can then Abel invert the noise-corrupted profile and compare it with the assumed $\varepsilon(r)$ profile. Unfortunately, this method can be time-consuming if one must deal with many different profiles. For example, if one wishes to investigate the effect of injector-gas flow rate on the radial emission profile of an ICP, a series of dissimilar theoretical $\varepsilon(r)$ profiles needs to be estimated.

A possible alternative approach is to derive a confidence interval for the Abel-inverted profile directly from the measured lateral profile. It will then become straightforward to select the most reliable portion of the profile. It will also provide a statistical means for comparing two different Abel-inverted profiles; the degree of overlap of the confidence intervals can be employed just as in a statistical test of significance between two means.

There are two objectives of the present paper. The primary objective is to describe and evaluate a novel method to estimate the statistical confidence intervals for any Abel-inverted

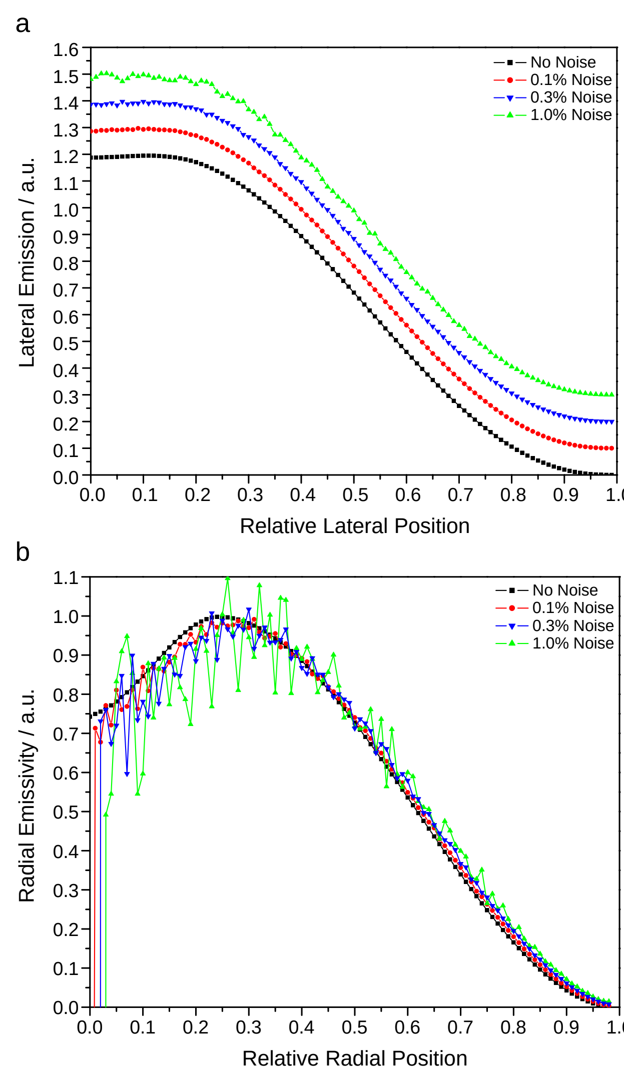

Fig. 2. (a) Lateral emission profile with 0%, 0.1%, 0.3% and 1% added noise. Each curve is offset from the nearest by 0.1 unit for clarity. (b) Abel-inverted radial emission profile from the lateral emission profiles in (a), with 0%, 0.1%, 0.3% and 1% added noise.