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ORIGINAL ARTICLE

Detuning effects in Haar wavelet spectrum of pulsed-driven qubit

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KEYWORDS

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Abstract The transient scattered radiation due to interaction of a short laser pulse (of rectangular shape) with a qubit is studied through the Haar wavelet window spectrum. Asymmetrical structure in the spectrum is shown due to frequency miss-match of the laser and qubit frequencies and the shift window parameter.

1. Introduction

Analyzing the scattered radiation due to the interaction of a single qubit (taken as spin-$\frac{1}{2}$ or 2-level atom system) with a short laser pulse is one of the many basic studies in signal information processing. Within a quantum framework the (transient) scattered radiation, known as fluorescent spectrum, is quantified as the Fourier transform (FT) of the auto-correlation function of the induced atomic dipoles (Mollow, 1969; Eberly and Wodkiewicz, 1977; Eberly, 1984). Transient florescent spectra of 2-level atomic systems (qubits) driven by different shapes of laser pulses (e.g., rectangular, triangular, etc. Newbold and Saloma, 1980; Rzazewski and Florjanczyk, 1984; Florjanczyk et al., 1985; Haus et al., 1984; Rodgers and Swain, 1991; Joshi and Hassan, 2002; Hassan et al., 2008, 2010) have been studied in detail.

In the usual Fourier analysis, a signal is represented as sum of sinusoidal functions of various frequencies. This is suitable for signals that change very slowly with time or for very noisy signals that change regularly with time (David, 2000; Addison, 2004). For non-stationary signals and signals with sudden change, Fourier analysis averaging over the entire length of the signal and hence the fixed time-width windowed FT has its limitation, as some ‘‘fine’’ detail is lost. Such ‘‘fine detail’’ or time localization analysis of different frequency components of a given signal is called wavelet transform (Croca, 2003). With windowed wavelet transform, the time width is adjusted to the frequency in such a way that high frequency wavelets will be narrow and vice-versa (Feruandez and Rojas, 2002). For resonant rectangular pulsed-driven qubit models, wavelet transform of the scattered radiation has been examined with different window functions: Morlet’s and Mexican hat wavelets (Mohamed et al., 2007) and Haar wavelet (Hassan et al., 2011).

In the present work, we examine the transient fluorescent Haar wavelet spectrum of a single qubit excited by a non-resonant rectangular laser pulse and compare the results with the corresponding ones for the resonant case (Hassan et al., 2011). The paper is presented as follows in Section 2, we present the essential analytical formulas for the Haar wavelet spectrum, followed by discussion of computational plots in Section 3. A summary is given in Section 4.

2. Haar wavelet transient spectrum

For a single qubit excited by a short laser pulse, the general form of the transient scattered radiation is given by (Eberly and Wodkiewicz, 1977; Rodgers and Swain, 1991).

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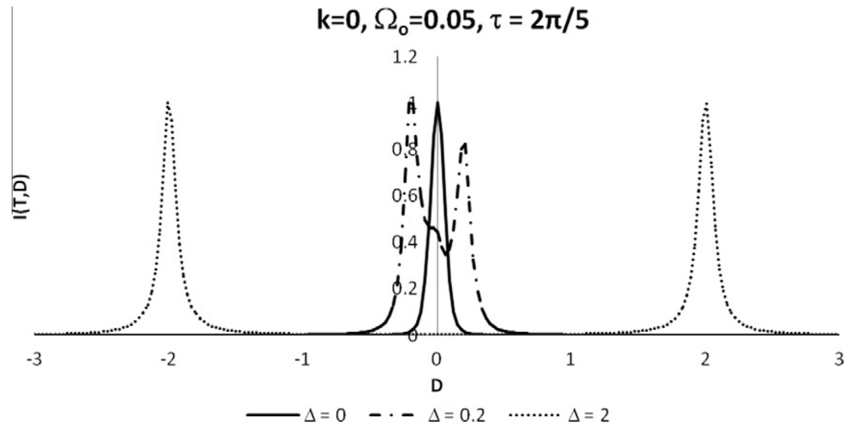

Figure 1a Transient Haar wavelet normalized spectrum $I(\tau, D)$ verses the filter's detuning parameter D for $\Omega_0 = 0.05$, $\tau = 2\pi/5$, $k = 0$ and various values of atomic detuning $\Delta = 0{,}0.2{,}2$.

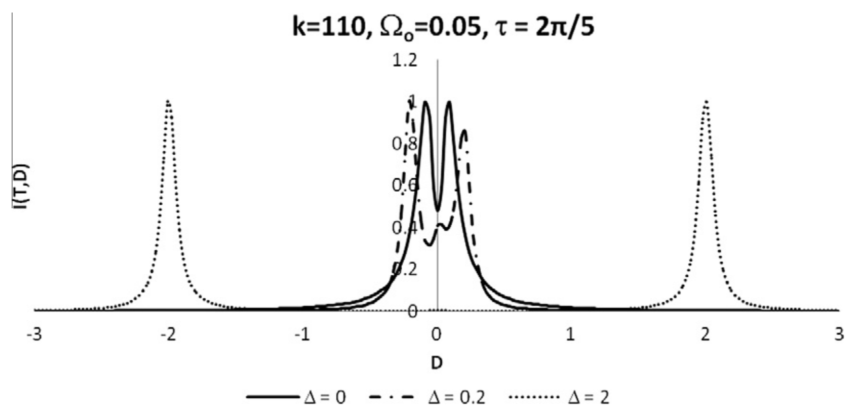

Figure 1b As Fig. 1a but for $k = 110$.

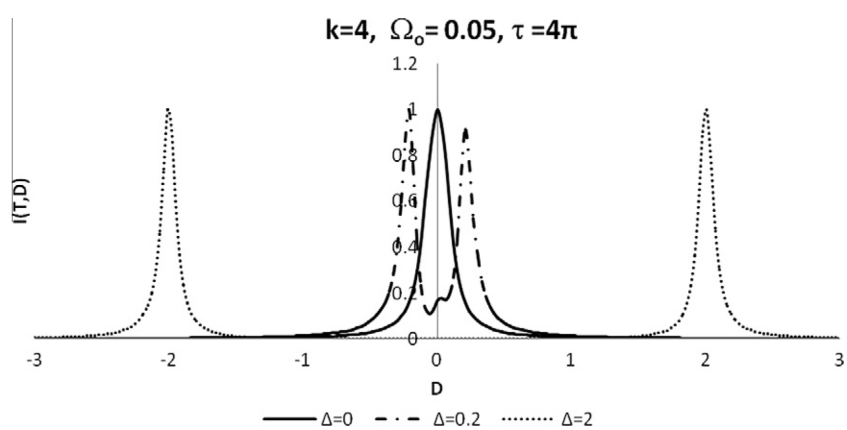

Figure 1c As Fig. 1a but for $k = 4$, $\tau = 4\pi$.

$$S(t, \omega_f, \Gamma) = 2\Gamma \int_{-\infty}^{t} dt_1 \int_{-\infty}^{t} dt_2 H(t - t_1) H^*(t - t_2) \times \langle \widehat{S}_+(t_1) \widehat{S}_-(t_2) \rangle \tag{1}$$

Where $\widehat{S}_+(t_1)\widehat{S}_-(t_2)$ is the quantum average of the atomic dipole–dipole spin auto-correlation function with $\widehat{S}_\pm$ are the atomic spin-up (down) operators and $H(t)$ is the (filter) window function of the radiation detector. For the case of narrow Lorentzian filter, it has the form

$$H_F(t) = e^{i\omega_f t} e^{-\Gamma|t|} \tag{2}$$

Where ω_f and Γ are the central frequency and width of the filter, respectively. With the narrow window function (2) the