Journal of the Association of Arab Universities for Basic and Applied Sciences (2014) **16**, 11–15

University of Bahrain

Journal of the Association of Arab Universities for Basic and Applied Sciences

www.elsevier.com/locate/jaaubas
www.sciencedirect.com

ORIGINAL ARTICLE

A modified analytical technique for Jeffery–Hamel flow using sumudu transform

Sushila [a,*], Jagdev Singh [b], Y.S. Shishodia [c]

[a] *Department of Physics, Jagan Nath University, Rampura, Chaksu, Jaipur 303901, Rajasthan, India*
[b] *Department of Mathematics, Jagan Nath University, Rampura, Chaksu, Jaipur 303901, Rajasthan, India*
[c] *Jagan Nath University, Rampura, Chaksu, Jaipur 303901, Rajasthan, India*

Received 30 May 2013; revised 19 August 2013; accepted 19 October 2013
Available online 18 November 2013

KEYWORDS

Jeffery–Hamel flow;
Homotopy perturbation method;
Sumudu transform;
Fluid mechanics;
Nonlinear equation

Abstract The main objective of this paper is to present a reliable approach to compute an approximate solution of Jeffery–Hamel flow by using the modified homotopy perturbation method coupled with sumudu transform. The method finds the solution without any discretization or restrictive assumptions and avoids the round-off errors. The fact that this technique solves nonlinear problems without using Adomian's polynomials can be considered as a clear advantage of this algorithm over the decomposition method. The numerical solutions obtained by the proposed method indicate that the approach is easy to implement and computationally very attractive.

1. Introduction

Internal flow between two plates is one of the most applicable cases in mechanics, civil and environmental engineering. In simple cases, the one-dimensional flow through tube and parallel plates, this is known as Couette–Poisseuille flow, has an exact solution, but in general, like most of fluid mechanic equations, a set of nonlinear equations must be solved which make some problems for analytical solution.

The flow between two planes that meet at an angle was first analyzed by Jeffery (1915) and Hamel et al. (1916) and so, it is known as Jeffery–Hamel flow, too. They worked mathematically on incompressible viscous fluid flow through convergent-divergent channels. They presented an exact similarity solution of the Navier–Stokes equations. In the special case of two-dimensional flow through a channel with inclined plane walls meeting at a vertex and with a source or sink at the vertex and have been studied extensively by several authors and discussed in many textbooks e.g. (Rosenhead, 1940; White, 1991; Esmali et al., 2008; Joneidi et al., 2010; Ganji et al., 2009; Inc et al., 2013). Sadri (1997) has denoted that Jeffery–Hamel is used as a asymptotic boundary condition to examine a steady two-dimensional flow of a viscous fluid in a channel. But, here some symmetric solutions of the flow have been considered, although asymmetric solutions are both possible and of physical interest (Sobey and Drazin, 1986).

Most of the scientific problems such as Jeffery–Hamel flow and other fluid mechanic problems are inherently nonlinear. Except a limited number of these problems, most of them do not have an exact solution. There exists a wide class of literature dealing with the problems of approximate solutions to nonlinear equations with various different methodologies,

* Corresponding author. Tel.: +91 9461673365.
E-mail addresses: sushila@jagannathuniversity.org, sushila.jag@gmail.com (Sushila), jagdevsinghrathore@gmail.com (J. Singh), yad.shi@gmail.com (Y.S. Shishodia).

Peer review under responsibility of University of Bahrain.

1815-3852 © 2013 Production and hosting by Elsevier B.V. on behalf of University of Bahrain.
http://dx.doi.org/10.1016/j.jaubas.2013.10.001

called perturbation methods. But, the perturbation methods have some limitations e.g., the approximate solution involves a series of small parameters which poses difficulty since majority of nonlinear problems have no small parameters at all. Although appropriate choices of small parameters some times lead to an ideal solution, in most of the cases unsuitable choices lead to serious effects in the solutions. Therefore, an analytical method is welcome which does not require a small parameter in the equation modeling the phenomenon. The homotopy perturbation method (HPM) was first introduced and developed by He (1999, 2005, 2006a, 2006b, 2012). It was shown by many authors that this method provides improvements over existing numerical techniques (Ganji and Ganji, 2008; Ganji et al., 2008, 2009; Rashidi et al., 2009; Yildirim and Sezer, 2010; Noor et al., 2013; Mirzabeigy et al., 2013). In recent years, many authors have paid attention to study the solutions of linear and nonlinear partial differential equations by using various methods combined with the Laplace transform (Khuri, 2001; Khan et al., 2012; Gondal and Khan, 2010; Singh et al., 2013a) and sumudu transform (Singh et al., 2011, 2013b).

In this paper, we present a modified analytical technique namely the modified homotopy perturbation method (MHPM) coupled with sumudu transform to obtain the approximate solution of nonlinear equation governing Jeffery–Hamel flow. The MHPM coupled with sumudu transform provides the solution in a rapid convergent series which may lead to the solution in a closed form. The advantage of this method is its capability of combining two powerful methods for obtaining exact and approximate solutions for nonlinear equations.

2. Sumudu transform

In early 90's, Watugala (1993) introduced a new integral transform, named the sumudu transform and applied it to the solution of ordinary differential equation in control engineering problems. The sumudu transform is defined over the set of functions

$$\mathrm{A} = \{f(t) | \exists \mathrm{M}, \tau_1, \tau_2 > 0, |\mathrm{f}(t)| < \mathrm{M}\, \mathrm{e}^{|t|/\tau_j}, \text{if } t \in (-1)^j \times [0, \infty)\}$$

by the following formula

$$\bar{\mathrm{f}}(\mathrm{u}) = \mathrm{S}[\mathrm{f}(t)] = \int_0^\infty \mathrm{f}(\mathrm{u}t)\mathrm{e}^{-t}\mathrm{d}t, \mathrm{u} \in (-\tau_1, \tau_2). \tag{1}$$

Some of the properties of the sumudu transform were established by Asiru (2001). Further, fundamental properties of this transform were established by Belgacem et al. (2003), Belgacem and Karaballi (2006), Belgacem (2006). In fact it was shown that there is a strong relationship between sumudu and other integral transform, see Kilicman et al. (2011). In particular the relation between sumudu transform and Laplace transforms was proved in Kilicman and Eltayeb (2010). The sumudu transform has scale and unit preserving properties, so it can be used to solve problems without resorting to a new frequency domain.

3. Mathematical model

Consider the steady unidirectional flow of an incompressible viscous fluid flow from a source or sink at the intersection between two rigid plane walls that the angle between them is 2α as it is shown in Fig. 1.

The velocity is assumed only along radial direction and depends on r and θ. Conservation of mass and momentum for two-dimensional flow in the cylindrical coordinate can be expressed as the following (Schlichting, 2000)

$$\frac{1}{r}\frac{\partial}{\partial r}(rU_r) + \frac{1}{r}\frac{\partial}{\partial r}(rU_\theta) = 0, \tag{2a}$$

$$\rho\left(U_r\frac{\partial U_r}{\partial r} + \frac{U_\theta}{r}\frac{\partial U_r}{\partial \theta} - \frac{U_\theta^2}{r}\right) = -\frac{\partial P}{\partial r} + \frac{1}{r}\frac{\partial(r\tau_{rr})}{\partial r} + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial r} - \frac{\tau_{r\theta}}{r}, \tag{2b}$$

$$\rho\left(U_r\frac{\partial U_\theta}{\partial r} + \frac{U_\theta}{r}\frac{\partial U_\theta}{\partial \theta} - \frac{U_r U_\theta}{r}\right) = -\frac{1}{r}\frac{\partial P}{\partial \theta} + \frac{1}{r^2}\frac{\partial(r\tau_{r\theta})}{\partial r} + \frac{1}{r}\frac{\partial \tau_{\theta\theta}}{\partial \theta} - \frac{\tau_{r\theta}}{r}, \tag{2c}$$

where P is the pressure term, U_r and U_θ are the velocities in r and θ directions, respectively. Stress components are defined as follows:

$$\tau_{rr} = \mu\left(2\frac{\partial U_r}{\partial r} - \frac{2}{3}div(\vec{U})\right), \tag{3a}$$

$$\tau_{\theta\theta} = \mu\left(2\left(\frac{1}{r}\frac{\partial U_\theta}{\partial \theta} + \frac{U_r}{r}\right) - \frac{2}{3}div(\vec{U})\right), \tag{3b}$$

$$\tau_{r\theta} = \mu\left(2\frac{\partial}{\partial r}\left(\frac{U_\theta}{r}\right) + \frac{1}{r}\left(\frac{\partial U_r}{\partial \theta}\right)\right). \tag{3c}$$

Considering $U_\theta = 0$ for purely radial flow leads to continuity and Navier–Stokes equations in polar coordinates become

$$\frac{\rho}{r}\frac{\partial}{\partial r}(rU_r) = 0, \tag{4a}$$

$$U_r\frac{\partial U_r}{\partial r} = -\frac{1}{\rho}\frac{\partial P}{\partial r} + \upsilon\left[\frac{\partial^2 U_r}{\partial r^2} + \frac{1}{r}\frac{\partial U_r}{\partial r} + \frac{1}{r^2}\frac{\partial^2 U_r}{\partial \theta^2} - \frac{U_r}{r^2}\right], \tag{4b}$$

$$-\frac{1}{\rho r}\frac{\partial P}{\partial \theta} + \frac{2\upsilon}{r^2}\frac{\partial U_r}{\partial \theta} = 0. \tag{4c}$$

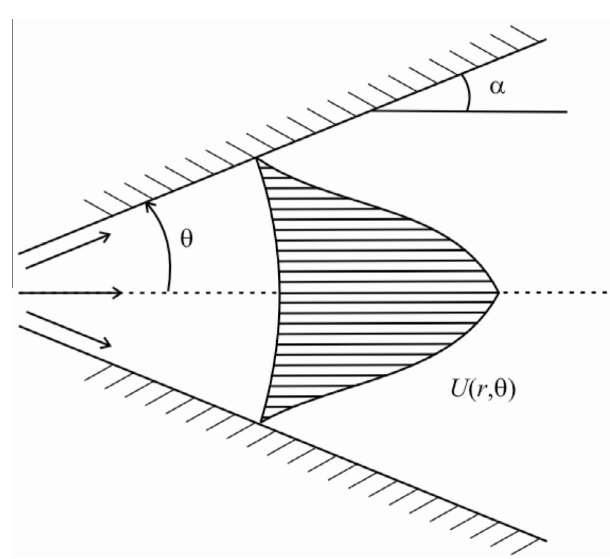

Figure 1 Schematic figure of the problem.