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ORIGINAL ARTICLE

Exact solutions of the (2 + 1)-dimensional cubic Klein–Gordon equation and the (3 + 1)-dimensional Zakharov–Kuznetsov equation using the modified simple equation method

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Abstract Exact solutions of nonlinear evolution equations (NLEEs) play a vital role to reveal the internal mechanism of complex physical phenomena. In this article, we implemented the modified simple equation (MSE) method for finding the exact solutions of NLEEs via the (2 + 1)-dimensional cubic Klein–Gordon (cKG) equation and the (3 + 1)-dimensional Zakharov–Kuznetsov (ZK) equation and achieve exact solutions involving parameters. When the parameters are assigned special values, solitary wave solutions are originated from the exact solutions. It is established that the MSE method offers a further influential mathematical tool for constructing exact solutions of NLEEs in mathematical physics.

1. Introduction

Nonlinear phenomena exist in all areas of science and engineering, such as fluid mechanics, plasma physics, optical fibers, biology, solid state physics, chemical kinematics, chemical physics and so on. It is well known that many NLEEs are widely used to describe these complex physical phenomena. Therefore, research to look for exact solutions of NLEEs is extremely crucial. So, to find effective methods to discover analytic and numerical solutions of nonlinear equations have drawn an abundance of interest by a diverse group of researchers. Accordingly, they established many powerful and efficient methods and techniques to explore the exact traveling wave solutions of nonlinear physical phenomena, such as, the Hirota's bilinear transformation method (Hirota, 1973; Hirota and Satsuma, 1981), the tanh-function method (Malfliet, 1992; Nassar et al., 2011), the (G'/G)-expansion method (Wang et al., 2008; Zayed, 2010; Zayed and Gepreel, 2009; Akbar et al., 2012a,b,c,d; Akbar and Ali, 2011a; Shehata, 2010), the

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Exp-function method (He and Wu, 2006; Akbar and Ali, 2011b; Naher et al., 2011, 2012), the homogeneous balance method (Wang, 1995; Zayed et al., 2004), the F-expansion method (Zhou et al., 2003), the Adomian decomposition method (Adomian, 1994), the homotopy perturbation method (Mohiud-Din, 2007), the extended tanh-method (Abdou, 2007; Fan, 2000), the auxiliary equation method (Sirendaoreji, 2004), the Jacobi elliptic function method (Ali, 2011), Weierstrass elliptic function method (Liang et al., 2011), modified Exp-function method (He et al., 2012), the modified simple equation method (Jawad et al., 2010; Zayed, 2011a, Zayed and Ibrahim, 2012; Zayed and Arnous, 2012), the extended multiple Riccati equations expansion method (Gepreel and Shehata, 2012; Gepreel, 2011a; Zayed and Gepreel, 2011) and others (Gepreel, 2011b,c).

The objective of this article is to look for new study relating to the MSE method to examine exact solutions to the celebrated (2 + 1)-dimensional cKG equation and the (3 + 1)-dimensional ZK equations to establish the advantages and effectiveness of the method. The cKG equation is used to model many different nonlinear phenomena, including the propagation of dislocation in crystals and the behavior of elementary particles and the propagation of fluxions in Josephson junctions. The (3 + 1)-dimensional Zakharov–Kuznetsov equation describes weakly nonlinear wave process in dispersive and isotropic media e.g., waves in magnetized plasma or water waves in shear flows.

The article is organized as follows: In Section 2, the MSE method is discussed. In Section 3 we exert this method to the nonlinear evolution equations pointed out above, in Section 4 physical explanation, in Section 5 comparisons and in Section 6 conclusions are given.

2. The MSE method

Suppose the nonlinear evolution equation is in the form,

$$F(u, u_t, u_x, u_y, u_z, u_{xx}, u_{tt}, \ldots) = 0, \tag{2.1}$$

where F is a polynomial of $u(x, y, z, t)$ and its partial derivatives wherein the highest order derivatives and nonlinear terms are involved. The main steps of the MSE method (Jawad et al., 2010; Zayed, 2011a; Zayed and Ibrahim, 2012; Zayed and Arnous, 2012) are as follows:

Step 1: The traveling wave transformation,

$$u(x, y, z, t) = u(\xi), \qquad \xi = x + y + z \pm \lambda t \tag{2.2}$$

allows us to reduce Eq. (2.1) into the following ordinary differential equation (ODE):

$$P(u, u\prime, u\prime\prime, \ldots) = 0, \tag{2.3}$$

where P is a polynomial in $u(\xi)$ and its derivatives, while $u\prime(\xi) = \frac{du}{d\xi}$.

Step 2: We suppose that Eq. (2.3) has the solution in the form

$$u(\xi) = \sum_{i=0}^{n} C_i \left(\frac{\phi\prime(\xi)}{\phi(\xi)} \right)^i, \tag{2.4}$$

where $C_i (i = 0, 1, 2, 3, \ldots)$ are arbitrary constants to be determined, such that $C_n \neq 0$ and $\phi(\xi)$ is an unspecified function to be found out afterward.

Step 3: We determine the positive integer n appearing in Eq. (2.4) by considering the homogeneous balance between the highest order derivatives and the highest order nonlinear terms come out in Eq. (2.3).

Step 4: We substitute Eq. (2.4) into (2.3) and then we account the function $\phi(\xi)$. As a result of this substitution, we get a polynomial of $(\phi'(\xi)/\phi(\xi))$ and its derivatives. In this polynomial, we equate the coefficients of same power of $\phi^{-j}(\xi)$ to zero, where $j \geqslant 0$. This procedure yields a system of equations which can be solved to find $C_i, \phi(\xi)$ and $\phi'(\xi)$. Then the substitution of the values of C_i, $\phi(\xi)$ and $\phi'(\xi)$ into Eq. (2.4) completes the determination of exact solutions of Eq. (2.1).

3. Applications

3.1. The (2 + 1)-dimensional cubic Klein–Gordon (cKG) equation

In this sub-section, first we will exert the MSE method to find the exact solutions and solitary wave solutions of the celebrated (2 + 1)-dimensional cKG equation,

$$u_{xx} + u_{yy} - u_{tt} + \alpha u + \beta u^3 = 0 \tag{3.1}$$

where α and β are non zero constants.

The traveling wave transformation

$$u = u(x, y, t), \quad \xi = x + y - \lambda t, \quad u(x, y, t) = u(\xi), \tag{3.2}$$

transforms the Eq. (3.1) to the following ODE:

$$(2 - \lambda^2) u\prime\prime + \alpha u + \beta u^3 = 0. \tag{3.3}$$

Balancing the highest order derivative and nonlinear term of the highest order, yields $n = 1$.

Thus, the solution Eq. (2.4) takes the form,

$$u(\xi) = C_0 + C_1 \left(\frac{\phi\prime(\xi)}{\phi(\xi)} \right), \tag{3.4}$$

where C_0 and C_1 are constants such that $C_1 \neq 0$, and $\phi(\xi)$ is an unspecified function to be determined. It is simple to calculate that

$$u\prime = C_1 \left(\frac{\phi\prime\prime}{\phi} - \left(\frac{\phi\prime}{\phi} \right)^2 \right), \tag{3.5}$$

$$u\prime\prime = C_1 \left(\frac{\phi\prime\prime\prime}{\phi} \right) - 3C_1 \left(\frac{\phi\prime\prime\phi\prime}{\phi^2} \right) + 2C_1 \left(\frac{\phi\prime}{\phi} \right)^3, \tag{3.6}$$

$$u^3 = C_1^3 \left(\frac{\phi\prime}{\phi} \right)^3 + 3C_1^2 C_0 \left(\frac{\phi\prime}{\phi} \right)^2 + 3C_1 C_0^2 \left(\frac{\phi\prime}{\phi} \right) + C_0^3. \tag{3.7}$$

Substituting the values of u, u'', and u^3 into Eq. (3.3) and equating the coefficients of $\phi^0, \phi^{-1}, \phi^{-2}, \phi^{-3}$ to zero, yields

$$\alpha C_0 + \beta C_0^3 = 0. \tag{3.8}$$

$$(\lambda^2 - 2)\phi''' - \left(\alpha + 3\beta C_0^2\right)\phi\prime = 0, \tag{3.9}$$

$$(\lambda^2 - 2)\phi\prime\prime + \beta C_0 C_1 \phi' = 0, \tag{3.10}$$

$$\left(\beta C_1^3 - 2(\lambda^2 - 2) C_1\right)(\phi')^3 = 0. \tag{3.11}$$

Solving Eq. (3.8), we obtain