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Folding of chaotic fractal space time

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Abstract

In this article, we introduce types of foldings in chaotic special types of space time. The effect of the foldings on the pure chaotic special space time is obtained. The limits of the foldings of special space time are achieved. The folding restricted on the charge and the mass are presented. The variations of the dimension of non-Riemannian chaotic manifold under the limit of folding are deduced. Types of the fractal foldings of the chaotic special space time are presented. Some applications concerning these relations are presented.

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1. Introduction and definitions

As is well known, the theory of folding is always one of interesting topics in Euclidian and Non-Euclidian space and it has been investigated from the various viewpoints by many branches of topology and differential geometry [5–24].

Deterministic chaos is now part of most scientific research, in varied fields. A big theory of everything has been discovered. Mathematicianes, physicists, and even biologists now have a common ground to work together. Chaos has revolutionized the way scientists think of nature, and the world around us [3,4,31]. Devaney describes a system as chaotic if there is (i) sensitivity to initial conditions, (ii) topological transitivity, and (iii) density of periodic points. He says that mathematical definitions approximate the idea of chaos, but do not capture it. The modern study of chaos began with the realization in 1960s that quit simple mathematical equations could model systems that were very complex. The simplest systems can produce extraordinarily difficult problems of predictability. Order may arise spontaneously in a system and from chaos spontaneous order which is called selforganized criticality [4,11,13,16]. Fractal is the geometry and patterns of deterministic chaos. For many the word fractal brings to mind the aesthetically pleasing, colorful images that are generated through computer graphics. Mandelbrott used the word to refer to fractional dimension [32]. The

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dimension of Euclidean geometry are 0, 1, 2, and 3 which are integers from the set $\{\ldots, -3, -2, -1, 1, 2, 3, \ldots\}$. The point in Euclidean geometry represents no or zero dimensions, the line represnts one dimension, the plane represents two dimensions, and space represents three dimensions. Fractal dimension is a fractional or partial dimension such as 1.45, which lies between the integers 1 and 2 [32].

We adapted these definitions and concepts to fit coherently into the framework of this article.

Definition 1. For Riemannian manifolds M and N (not necessarily of the same dimension) a map $\phi{:}M \to N$ is said to be a topological folding of M into N if, for each piecewise geodesic path $\gamma{:}I \to M$ ($I = [0,1] \subseteq R$), the induced path $\phi o \gamma{:}I \to N$ is a piecewise geodesic. If, in addition, $\phi{:}M \to N$ preserves lengths of paths, we call $\phi{:}M \to N$ an isometric folding of M into N. If the continuous map $\phi{:}M \to N$ is a folding then $dimM \leq dimN$. Many types of foldings are discussed in [1,5,9,14,17,19,23]. Some applications are disscussed in [6,8,16,30].

Definition 2. If X is a topological space and $A \subset X$, there exist the continuous map r such that $r{:}X \to A$, $r(a) = a \forall a \in A$, So r is retraction function [2,22,25–27]. Many types of retractions are presented in [7,10,12,18].

Definition 3. A chaotic special space time B_i is the special space time B_i carries many physical characters. Each character represents special space time B_i homeomorphic to the orginal one [7,14,15].

2. Space time

The energy distribution of a static spherically symmetric charged black hole which the line element representing this special space time by [14,19,28,29].

$$dS^2 = \left(1 - \frac{2M}{r}\right) dt^2 - \left(1 - \frac{2M}{r}\right)^{-1} dr^2 - \left(1 - \frac{\alpha}{r}\right) r^2 (d\theta^2 + \sin^2 \theta d\Phi^2) \tag{1}$$

where $\alpha = Q^2(\exp(-2\Phi_o)/M)$, M and Q are respectively mass and charged parameters; Φ_o is the asymptotic value of dilation field.

3. Main results

In what follows we would like to introduce types of conditional foldings of chaotic special space time which the metric is defined as:

$$dS_i^2 = \left(1 - \frac{2M_i}{r(\eta)}\right) dt^2(\eta) - \left(1 - \frac{2M_i}{r(\eta)}\right)^{-1} dr^2(\eta) - \left(1 - \frac{\alpha_i}{r(\eta)}\right) r^2(\eta)(d\theta^2(\eta) + \sin^2 \theta(\eta) d\varphi^2(\eta)) \tag{2}$$

where $t(\eta)$, $r(\eta)$,$\theta(\eta)$ and $\phi(\eta)$ are functions of energy distribution, also $i = 0, 1, 2, \ldots \infty$, if $i = 0$ it is the space time. If $i = 1, 2, \ldots \infty$ then dS_i^2 represents the special pure chaotic ones. Also, $\alpha_o = Q_0^2(\exp(-2\phi_o)/M_o)$, $\alpha_1 = Q_1^2(\exp(-2\phi_o))/M_1, \ldots \alpha_\infty = Q_\infty^2(\exp(-2\phi_o))/M_\infty$. Moreover, the coordinate of the chaotic special space time are:

$$\chi_{i1} = \sqrt{\left(1 - \frac{2M_i}{r(\eta)}\right) t^2(\eta) + C_1}$$

$$\chi_{i2} = \sqrt{C_2 - (r^2(\eta) + 4M_i r(\eta) + 3M_i^2 \ln(r(\eta) - 2M_i))}$$

$$\chi_{i3} = \sqrt{C_3 - \left(1 - \frac{\alpha_i}{r(\eta)}\right) r^2(\eta)\theta^2(\eta)}$$

$$\chi_{i4} = \sqrt{C_4 - \left(1 - \frac{\alpha_i}{r(\eta)}\right) r^2(\eta) \sin^2 \theta(\eta) \phi^2(\eta)}$$

where C_1, C_2, C_3, and C_4 are the constant of integration.