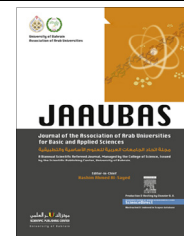




University of Bahrain
**Journal of the Association of Arab Universities for
Basic and Applied Sciences**

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ORIGINAL ARTICLE

Variational Iteration Method for the Solution of Seventh Order Boundary Value Problems using He's Polynomials



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Received 9 October 2013; revised 22 February 2014; accepted 9 March 2014

Available online 4 April 2014

KEYWORDS

Variational iteration method;
Homotopy perturbation
method;
Boundary value problems;
Linear and nonlinear
problems;
Approximate solution

Abstract The induction motor behaviour is represented by a fifth order differential equation model. Addition of a torque correction factor to the model accurately reproduces the transient torques and instantaneous real and reactive power flows of the full seventh order differential equation model. The variational iteration method using He's polynomials is employed to solve the seventh order boundary value problems. The approximate solutions to the problems are obtained in terms of a rapidly convergent series. Several numerical examples are given to illustrate the implementation and the efficiency of the method.

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Introduction

The theory of seventh order boundary value problems is not much available in the numerical analysis literature. These problems generally arise in modelling induction motors with two rotor circuits.

The induction motor behaviour is represented by a fifth order differential equation model. This model contains two stator state variables, two rotor state variables and one shaft speed. Normally, two more variables must be added to account for the effects of the second rotor circuit representing deep bars, a starting cage or rotor distributed parameters.

To avoid the computational burden of additional state variables when additional rotor circuits are required, model is often limited to the fifth order and rotor impedance is algebraically altered as a function of rotor speed under the assumption that the frequency of rotor current depends on the rotor speed. This approach is efficient for the steady state response with sinusoidal voltage, but it does not hold up during the transient conditions, when rotor frequency is not a single value. So the behaviour of such models show up in the seventh order (Richards and Sarma, 1994).

J. He was first to propose a new kind of analytical method for a non-linear problem called the variational iteration method. In (He, 1999a) J. He used variational iteration method to give approximate solutions for some well-known non-linear problems. In variational iteration method, the problems are initially approximated with possible unknown. Then a correction functional is constructed by a general Lagrange

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Peer review under responsibility of University of Bahrain.

multiplier, which can be identified optimally via the variational theory.

Homotopy perturbation method was also proposed by He (1999b, 2000b, 2006). To investigate the given problem with the help of homotopy perturbation method, firstly a homotopy equation is constructed. It is assumed that the solution of the problem is $u = \sum_{i=0}^{\infty} u_i c^i$. Substituting the value of u in the homotopy equation and equating the like powers of c , a system of differential equations is obtained. The corresponding solution of the system provides a series solution. The results revealed that the homotopy perturbation method is a powerful and accurate method for finding solutions for BVPs in the form of analytical expressions and presents a rapid convergence for the solutions.

Jafari et al. (Jafari et al., 2011) proposed the homotopy analysis method to solve an evolution equation. The authors compared the results obtained with the help of homotopy analysis method and the results obtained with the help of Adomian decomposition method. Siddiqi and Iftikhar (Siddiqi and Iftikhar, 2013a) used the variation of parameter method to solve the seventh order boundary value problems. In (Siddiqi and Iftikhar, 2013b) the authors used the homotopy analysis method, an approximating technique for solving linear and nonlinear higher order boundary value problems.

Odibat discussed the convergence of variational iteration method in (Odibat, 2010). Tatari and Dehghan presented the sufficient conditions to guarantee the convergence of the variational iteration method (Tatari and Dehghan, 2007).

The aim of this study is to solve the seventh order boundary value problems and the variational iteration method using He's polynomials is used for this purpose.

Variational Iteration Method using He's Polynomials (He, 1999a)

The boundary value problem is considered as under

$$L[u(x)] + N[u(x)] = g(x), \quad (1)$$

where L and N are linear and nonlinear operators respectively and $g(x)$ is a forcing term. Following the variational iteration method used by J. He (He, 1998, 1999a,b, 2000a, 2001) the correct functional for the problem (1) can be written as follows

$$u_{n+1}(x) = u_n(x) + \int_0^x \lambda(t) \{Lu_n(t) + N\tilde{u}_n(t) - g(t)\} dt, \quad (2)$$

where λ is a Lagrange multiplier, that can be identified optimally via variational iteration method. Here, $\tilde{u}_n$ is considered to be a restricted variation which shows that $\delta\tilde{u}_n = 0$. Making the correct functional (2) stationary, yields

$$\begin{aligned} \delta v_{n+1}(x) &= \delta v_n(x) + \delta \int_0^x \lambda(t) \{Lv_n(t) + N\tilde{v}_n(t) - g(t)\} dt \\ &= \delta v_n(x) + \int_0^x \delta \{\lambda(t) Lv_n(t)\} dt. \end{aligned} \quad (3)$$

Its stationary conditions can be obtained using integration by parts in Eq. (3). Therefore, the Lagrange multiplier can be written as

$$\lambda = \frac{(-1)^m (t-x)^{m-1}}{(m-1)!}. \quad (4)$$

Applying the homotopy perturbation method (He, 1999b, 2000b, 2006), the following relation is obtained as follows

$$\begin{aligned} \sum_{i=0}^{\infty} p^i u_i(x) &= u_0(x) + \int_0^x \lambda(t) \left\{ L \left(\sum_{i=0}^{\infty} p^i u_i \right) + N \left(\sum_{i=0}^{\infty} p^i \tilde{u}_i \right) \right\} dt \\ &\quad - \int_0^x \lambda(t) g(t) dt, \end{aligned} \quad (5)$$

Equating the like powers of p gives $u_0, u_1, \dots$. The embedding parameter $p \in [0, 1]$ can be used as an expanding parameter. The nonlinear term can be expanded into He's polynomials (Ghorbani, 2009). The approximate solution of the problem (1); therefore, can be expressed as follows

$$u = \lim_{p \rightarrow 1} \sum_{i=0}^{\infty} p^i u_i = u_0 + u_1 + u_2 + \dots \quad (6)$$

The series (6) is convergent for most of the cases. It is assumed that (6) has a unique solution.

In fact, the solution of the problem (1) is considered as the fixed point of the following functional under the suitable choice of the initial term $v_0(x)$.

$$v_{n+1}(x) = v_n(x) + \int_0^x \lambda(t) \{Lv_n(t) + Nv_n(t) - g(t)\} dt. \quad (7)$$

Convergence

In this section, we will present Banach's theorem about the convergence of the variational iteration method using He's polynomials. The method changes the given differential equation into a recurrence sequence of functions. The limit of this sequence is considered as the solution of the given differential equation.

Theorem 1. (Banach's fixed point theorem) (Tatari and Dehghan, 2007) Suppose that X is a Banach space and $B : X \rightarrow X$ is a nonlinear mapping, and assume that

$$\|B[u] - B[\bar{u}]\| \leq \gamma \|u - \bar{u}\|, \quad \forall u, \bar{u} \in X. \quad (8)$$

for some constant $\gamma < 1$. Then B has a unique fixed point. Moreover, the sequence

$$u_{n+1} = B[u_n] \quad (9)$$

with an arbitrary choice of $u_0 \in X$ converges to the fixed point of B and

$$\begin{aligned} \|u_k - u_l\| &\leq \|u_k - u_{k-1}\| + \dots + \|u_{l+1} - u_l\| = \|B(u_{k-1}) \\ &\quad - B(u_{k-2})\| + \dots + \|B(u_l) - B(u_{l-1})\| \\ &\leq \gamma \|u_{k-1} - u_{k-2}\| + \dots + \gamma \|u_l - u_{l-1}\| \leq \dots \leq \\ &\leq (\gamma^{k-2} + \gamma^{k-3} + \dots + \gamma^{l-1}) \|u_1 - u_0\| \\ &\leq \frac{\gamma^{l-1}}{1-\gamma} \|u_1 - u_0\|, \end{aligned} \quad (10)$$

where, $\gamma < 1$, it can be assumed that $k > l \geq 1$. This yields $\|u_k - u_l\| \rightarrow 0$ as $k, l \rightarrow \infty$ and hence $(u_k)_{k=1}^{\infty}$ is a Cauchy sequence. Since X is a Banach space the sequence converges to a fixed point.

According to Theorem 1, for the nonlinear mapping

$$B[u] = u(x) + \int_0^x \lambda(t) \{Lu_n(t) + N\tilde{u}_n(t) - g(t)\} dt, \quad (11)$$

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