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ORIGINAL ARTICLE

The optimal homotopy asymptotic method with application to modified Kawahara equation

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Abstract In this paper the optimal homotopy asymptotic method (OHAM) is introduced for obtaining the approximate solution of modified Kawahara equations. The OHAM results are compared with Variational Iteration Method (VIM), Homotopy Perturbation Method (HPM) and Exact solutions. The comparison of OHAM with these methods reveals that OHAM is very effective, reliable and efficient.
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1. Introduction

The modified Kawahara equation has wide applications in physics such as plasma waves, capillary-gravity water waves, water waves with surface tension, shallow water waves (see Berloff and Howard, 1997; Hunter and Scheurle, 1998; Kawahara, 1972; Jin, 2009). The modified Kawahara equation has the form

$$\zeta_t + \zeta^2\zeta_x + r\zeta_{xxx} + q\zeta_{xxxxx} = 0, \quad (1)$$

where r, and q are nonzero real constants. The modified Kawahara equation has been solved by different analytic and numerical methods. These methods include the tanh-function method, extended tanh-function method, VIM, Sine–Cosine method, HPM, Jacobi elliptic function method and Adomian decomposition method (ADM) (see Sirendaoreji, 2004; Wazwaz, 2007; Yusufoglu and Bekir, 2006; Wazwaz, 2006; Bibi and Mahyuddin, 2014; Noor et al., 2013; Zhang, 2005; Polat et al., 2006). The perturbation methods containing a small parameter and are difficult to be found were used for the solution of nonlinear boundary value problems (BVPs) (see O'Malley, 1974; Cole, 1968; Liu, 1997). The homotopy perturbation methods such as HPM, HAM and Artificial parameter method (see Liu, 1997) were introduced for finding the small parameter. Recently Marinca and Herisanu proposed OHAM for the solution of nonlinear BVPs (see Marinca et al., 2008, 2009; Herisanu et al., 2008; Herisanu and Marinca, 2010a,b). The authors have applied OHAM for obtaining the approximate solutions of nonlinear BVPs (see Islam and Shah, 2010; Idrees et al., 2010, 2012; Ullah et al., 2014; Nawaz et al., 2013). After introducing this method the perturbation methods become independent of the assumption of small parameter.

The first part of paper is introduction, and part 2 is devoted to the analysis of the proposed method. In part 3 solution of modified Kawahara equation is presented by OHAM and absolute errors with respect to Exact solution. The 3D and 2D images of the approximate solutions and exact solution are given. In all cases, the proposed method yields very encouraging results.

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Table 1 Comparison of absolute errors of OHAM solution to VIM, HPM and Exact solution at $x = -5$ for various values of t.

t	OHAM Sol	VIM Sol	HPM Sol	Exact Sol	Abs Errors
0	0.000947498	0.000947498	0.000947498	0.000947498	1.08420×10^{-19}
2	0.000947498	0.000947498	0.000947498	0.000946363	1.13562×10^{-6}
4	0.000947498	0.000947498	0.000947498	0.000944851	2.64695×10^{-6}
6	0.000947498	0.000947498	0.000947498	0.000942967	4.53160×10^{-6}
8	0.000947498	0.000947498	0.000947498	0.000940712	6.78656×10^{-6}
10	0.000947498	0.000947498	0.000947498	0.000938090	9.40828×10^{-6}

Table 2 Comparison of absolute errors of OHAM solution to VIM, HPM and Exact solution at $x = -2.5$ for different values of t.

t	OHAM Sol	VIM Sol	HPM Sol	Exact Sol	Abs Errors
0	0.000948387	0.000948387	0.000948387	0.000948387	1.08420×10^{-19}
2	0.000948387	0.000948387	0.000948387	0.000947723	6.63492×10^{-7}
4	0.000948387	0.000948387	0.000948387	0.000946682	1.70487×10^{-6}
6	0.000948387	0.000948387	0.000948387	0.000945264	3.12248×10^{-6}
8	0.000948387	0.000948387	0.000948387	0.000943473	4.91406×10^{-6}
10	0.000948387	0.000948387	0.000948387	0.000941310	7.07676×10^{-6}

Table 3 Comparison of absolute errors of OHAM solution to VIM, HPM and Exact solution at $x = 0$ for various values of t.

t	OHAM Sol	VIM Sol	HPM Sol	Exact Sol	Abs Errors
0	0.000948387	0.000948683	0.000948683	0.000948683	0
2	0.000948387	0.000948683	0.000948683	0.000948494	1.89711×10^{-7}
4	0.000948387	0.000948683	0.000948683	0.000947925	7.58542×10^{-7}
6	0.000948387	0.000948683	0.000948683	0.000946978	1.70558×10^{-6}
8	0.000948387	0.000948683	0.000948683	0.000945654	3.02932×10^{-6}
10	0.000948387	0.000948683	0.000948683	0.000943956	4.72765×10^{-6}

Table 4 Comparison of OHAM solution to VIM, HPM and Exact solution at $x = 2.5$ for various values of t.

t	OHAM Sol	VIM Sol	HPM Sol	Exact Sol	Abs Errors
0	0.000948387	0.000948387	0.000948387	0.000948387	1.08420×10^{-19}
2	0.000948387	0.000948387	0.000948387	0.000948671	2.84543×10^{-7}
4	0.000948387	0.000948387	0.000948387	0.000948577	1.89683×10^{-7}
6	0.000948387	0.000948387	0.000948387	0.000948102	2.84430×10^{-7}
8	0.000948387	0.000948387	0.000948387	0.000947250	1.13704×10^{-6}
10	0.000948387	0.000948387	0.000948387	0.000946020	2.36678×10^{-6}

Table 5 Comparison of OHAM solution VIM, HPM and Exact solution at $x = 5.0$ for various values of t.

t	OHAM Sol	VIM Sol	HPM Sol	Exact Sol	Abs Errors
0	0.000948387	0.000948387	0.000948387	0.000948387	1.08420×10^{-19}
2	0.000948387	0.000948387	0.000948387	0.000948671	2.84543×10^{-7}
4	0.000948387	0.000948387	0.000948387	0.000948577	1.89683×10^{-7}
6	0.000948387	0.000948387	0.000948387	0.000948102	2.84430×10^{-7}
8	0.000948387	0.000948387	0.000948387	0.000947250	1.13704×10^{-6}
10	0.000948387	0.000948387	0.000948387	0.000946020	2.36678×10^{-6}

2. Fundamental theory of OHAM

Consider the partial differential equation of the form

$$\mathcal{L}(\zeta(x,t)) + \mathcal{N}(\zeta(x,t)) + g(x,t) = 0, x \in \Omega \qquad (2)$$
$$\mathcal{B}(\zeta, \partial\zeta/\partial t) = 0,$$

where $\mathcal{L}$ is a linear operator and $\mathcal{N}$ is nonlinear operator. $\mathcal{B}$ is boundary operator, $\zeta(x,t)$ is an unknown function, and x and t denote spatial and time variables, respectively, Ω is the problem domain and $g(x,t)$ is a known function.

Using the basic idea of OHAM, the optimal homotopy $\psi(x,t;a):\Omega \times [0,1] \to R$ is constructed which satisfies the following condition.

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