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Statistical analysis for stochastic radiation transfer in finite participating media with Rayleigh scattering

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Abstract

The stochastic radiation transfer problem is studied in a finite participating planar continuously random medium. The problem is considered for specular-reflecting boundaries with Rayleigh scattering. The importance Rayleigh scattering arises in the atmospheric applications in the form of radiative transfer through clouds or in neutron transport with a quadratic formula of scattering. Such cluttered media should be analyzed in a statistical sense. As a result, the random variable transformation (RVT) technique is used to obtain the average of the solution functions that are represented by the probability-density function (PDF) of the solution process. The transport equation is solved deterministically to obtain a closed form of the solution as a function of optical depth x and optical thickness L. The solution is used to obtain the PDF of the solution functions by applying the RVT technique among the input random variable (L) and the output process (the solution functions). The obtained averages of the solution functions are used to obtain the complete analytical averages for some interesting physical quantities, namely, reflectivity and transmissivity, at the medium boundaries. The numerical results are calculated and represented graphically for different probability distribution functions.

Keywords: Stochastic integro-differential radiation transfer equation; Finite participating atmospheric random medium; Rayleigh scattering; RVT technique; Specular-reflecting boundaries; Probability distribution functions

1. Introduction

Obviously, a real medium requires statistical investigations to emphasize its inhomogeneities. For example, modern atmospheric models do not include the macroscopic geometry of atmospheric media. The most common approximation of partial randomness involves the fluctuations of the medium properties, which is a convenient way of describing its optical and radiative-transfer properties. Stochastic theory is an important approach, with radiative transfer in a stochastic media being a highly active research field in recent years [1–4].

The solution of a stochastic radiative transfer problem can be obtained when the probability distribution function (PDF) of the solution can be evaluated. This evaluation can be achieved by many methods and techniques, for example, the transformation technique [5], Wiener-Hermite expansion [6], stochastic linearization [7], and stochastic finite element method [8]. In the transformation technique [9], the PDF of the solution process is computed whenever the solution

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function can be represented in a closed form in the input variables. In this case, it can be said that we have a one-to-one mapping between the solution output and the random inputs, with a condition that the corresponding Jacobian of transformation can be evaluated. The use of this technique is, however, limited according to the conditions that should be satisfied for the existence of the random variable transformation (RVT) [10]. Very recently, the stochastic neutron transport in a fluctuating reactor was studied [4], where the solution inverse (and hence the Jacobian) was estimated easily for the case of semi-infinite media. Due to the difficulties involved in obtaining one-to-one mapping in the case of a finite planar media, we attempted to overcome these difficulties to obtain the PDF of the solution functions in this work.

Generally, stochastic principles are investigated by using different techniques for different cases. For example, Markovian statistics [2] are used for solving discrete randomly radiative transfer problems; we solved many related problems (e.g., [11]). For the theory of fluctuations (continuous randomness), Gaussian statistics have been used in many previous papers [12,13] to obtain the average solution. What can be used for other probability distributions? To answer this question, in this work, we applied the RVT technique to obtain the average solution for any probability distribution, either for a half-space medium [4] or a finite slab plane-parallel medium (present work).

In this paper, the stochastic radiative transfer problem is analyzed for a participating finite planar cluttered medium. The problem is considered for specular-reflecting boundaries with Rayleigh scattering. The medium is assumed to be continuously stochastic, that is, the extinction function is a continuous random function of position. Consequently, the optical variable x and optical thickness L of the medium are continuously random variables. To obtain the average solution over the medium fluctuations, a solution algorithm used that is mainly s based on using the RVT technique [4,9]. Following this treatment, the PDF of the solution process is obtained, and then, one can calculate any statistical quantity related to the solution. In the context of a finite planar media, the obtained deterministic solution is a combination of two independent exponential functions. Hence, determining the inverse of the solution is an impossible task. For the case of a non-atomic mix, each exponential solution function can be treated separately to obtain the corresponding PDF (via RVT), and then, the average is readily obtained. Therefore, the combination of the two average solution functions results in the total analytical average expression for the reflectivity and transmissivity. The algorithm of the RVT technique is applied to obtain the average solutions for different types of PDF (exponential, normal Gaussian, gamma). The numerical results of the average reflectivity and transmissivity at the medium boundaries are obtained and displayed graphically.

2. Basic equations

The radiative transfer through a finite planar participating medium with anisotropic scattering [2] can be described by

$$\left(\mu\frac{\partial}{\partial x}+1\right)I(x,\mu)=\frac{\omega}{2}\int_{-1}^{1}P(\mu,\mu')I(x,\mu')d\mu',\qquad 0\le x\le L,\quad -1\le\mu\le+1 \tag{1}$$

where $I\,(x,\mu)$ is the radiation intensity, with the optical space variable x and angular variable μ (the direction cosine of the transferred radiation), and ω is the single scattering albedo of the medium.

The problem modelled by Eq. (1) is subjected to the specular reflective boundaries in the form

$$\left.\begin{aligned}I(0,\mu)&=F+\rho_1^s I(0,-\mu)\\ I(L,-\mu)&=\rho_2^s I(L,\mu)\end{aligned}\right\},\quad \mu\ge 0 \tag{2}$$

where F is the external incident flux on the upper boundary ($x=0$) and ρ_i^s ($i=1,2$) are the specular reflectivities of the boundaries. For Rayleigh scattering, the function $P\left(\mu,\mu'\right)$ takes the form [14]

$$P(\mu,\mu')=\frac{3}{8}\left[(3-\mu^2)+(3\mu^3-1)\mu'^2\right] \tag{3}$$

Deterministically, the Pomraning-Eddington approximation is used to solve the problem. This method expresses the angular intensity $I(x,\mu)$ in the form [15,16]

$$I(x,\mu)=E(x)\varepsilon(x,\mu)+F(x)O(x,\mu) \tag{4}$$

where $E(x)$ is the radiant energy and $F(x)$ is the radiative net flux, which are defined by

$$E(x)=\int_{-1}^{1}I(x,\mu)d\mu\quad\text{and}\quad F(x)=\int_{-1}^{1}\mu I(x,\mu)d\mu \tag{5}$$

and $\varepsilon(x,\mu)$ and $O(x,\mu)$ are even and odd functions, respectively, in μ and slowly vary in x. Therefore, the solution is obtained in the analytical form as

$$I(x,\mu)=\alpha_0\sum_{n=0}^{\infty}\alpha_1^n\left\{I_3\Gamma_-(\mu)e^{-\nu(x+2nL)}-I_4\Gamma_+(\mu)e^{\nu(x-2(n+1)L)}\right\} \tag{6}$$