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Polarized-radiation transfer in stochastic finite planar atmospheric media

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Abstract

In this work, we investigate the polarized-radiation transfer problem in a planar stochastic atmospheric medium. A solution is presented for an arbitrary extinction function, which was assumed to be continuous in position and to exhibit fluctuations about the Gaussian distributed mean. The joint probability distribution function of the integral transform of these Gaussian random functions was used to calculate the ensemble-averaged quantities, such as the reflectivity and transmissivity, for an arbitrary correlation function. A modified Gaussian probability distribution function was used to average the solution to exclude probable negative values of the optical space variable. The deterministic solutions for the total intensity and the difference function, which were used to describe the polarized radiation, were obtained using the Pomraning–Eddington technique. The numerical results were presented for the Gaussian and modified Gaussian probability density functions for different degrees of polarization.

Keywords: Polarized-radiation transfer; Stochastic atmospheric medium; Gaussian statistics

1. Introduction

It is well known that the scattering by particles (aerosols, cloud drops, crystals, molecules and atoms) induces polarization in incident unpolarized radiation beams or modifies the initial polarization. Hence, an exact treatment of light propagation in a medium composed of such scattering particles must include the state of radiation polarization both before and after scattering. Therefore, the study of polarized-radiation transfer in stochastic media is of great interest with regard to planetary atmospheres. The first formulation of such a problem allowing polarization was developed by Chandrasekhar [1] to describe the spatial variation of the four Stokes parameters: I, K, U, and V.

There has been an increasing interest in the use of polarization in imaging through cluttered media (stochastic media). Optical imaging through clouds and fog, and microwave detection in clutter can benefit from the additional information provided by the polarization characteristics [2]. Polarization pulse propagation in random media has also received attention, with most of these studies using Monte Carlo calculations [3].

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Radiative transfer computations of the Earth's atmospheres continue to become more important as the emphasis on remote sensing shifts towards understanding the microphysical properties of clouds. In addition to a better understanding of the nonlinear relationships between rainfall rates and satellite-observed radiances [4], a large interest has developed in sub-mm ice clouds studies [5,6]. To achieve these goals, significant attention is being given to the polarization effects from emission/scattering due to particles and surfaces. Simulations of polarized-radiation transfer can yield results on this subject [7–9].

The study of radiative transfer with polarization in a finite plane-parallel stochastic atmospheric medium (such as clouds and fog) is presented in this paper. It is reasonable to characterize the stochastic atmospheric medium in a statistical sense. This requires specifying the probability distribution of fluctuating properties via spatial and temporal correlations, etc. A stochastic transport class of problems arises when the environmental properties of the background medium are random functions of position and time that are in scales on the order of, or longer than, several mean free paths.

Throughout this work, the atmospheric medium is considered to be a continuously stochastic background. The extinction function of the medium is assumed to be a continuous random function of position, with fluctuations about the mean taken as being Gaussian distributed. The joint probability distribution function of these Gaussian random variables is used to calculate the ensemble-averaged quantities, such as the average reflectivity and transmissivity over the medium fluctuations, for an arbitrary correlation function. The Pomraning–Eddington approximation is employed to yield deterministic solutions for the total intensity and the difference function of the polarized radiation [8,9]. Then, the solution is averaged using a Gaussian joint probability distribution function [10,11]. However, a weakness of the Gaussian model is that the random variables that are physically constrained to be positive can in fact, from mathematical point of view, take on negative values, albeit with an exponentially decreasing probability. This can potentially be cause for concern when ensemble averages are considered, especially when the fluctuation amplitude is large. Hence, a modified Gaussian probability density function [11] is used to overcome this defect. Numerical results are presented for both the average reflectivity $\langle R\rangle$ and transmissivity $\langle T\rangle$ for different degrees of polarization and values of the scattering albedo.

2. Basic equations

Considering the Rayleigh scattering phase function with polarization, the scalar radiative transfer equation

$$\left(\mu\frac{\partial}{\partial x}+1\right)I(x,\mu)=\frac{\omega}{2}\int_{-1}^{1}P\left(\mu,\mu'\right)I\left(x,\mu'\right)d\mu' \tag{1}$$

should be replaced by the two coupled equations describing the transfer of linearly polarized radiation [1,12] as

$$\begin{aligned}\left(\mu\frac{\partial}{\partial x}+1\right)I(x,\mu)&\\=\frac{\omega}{2}\int_{-1}^{1}d\mu'\Big\{&\left[1+\frac{1}{2}P_2(\mu)P_2\left(\mu'\right)\right]I\left(x,\mu'\right)\\&-\frac{1}{2}P_2(\mu)\left[1-P_2\left(\mu'\right)\right]K\left(x,\mu'\right)\Big\}\end{aligned} \tag{2}$$

and

$$\begin{aligned}\left(\mu\frac{\partial}{\partial x}+1\right)K(x,\mu)&\\=\frac{\omega}{4}[1-P_2(\mu)]\int_{-1}^{1}d\mu'\Big\{&\left[1-P_2\left(\mu'\right)\right]K\left(x,\mu'\right)\\&-P_2\left(\mu'\right)I\left(x,\mu'\right)\Big\},\quad 0\le x\le L,\ -1\le\mu\le 1\end{aligned} \tag{3}$$

where I is the total intensity, $I=I_l+I_r$, and K is the difference function, $K=I_l-I_r$, which are the main parameters used to define the Stokes parameters, whereas the other two parameters vanish for linearly polarized radiation ($U=V=0$). The variable $P_2(\mu)$ represents a second-order Legendre polynomial $P_2(\mu)=0.5(3\mu^2-1)$.

The boundary conditions for the finite medium are assumed to be

$$\begin{aligned}&I(0,\mu)=\Gamma(\mu),\quad K(0,\mu)=\alpha^*(\mu)\Gamma(\mu)\\&I(L,-\mu)=0,\quad K(L,-\mu)=0\end{aligned} \tag{4}$$

where $\Gamma(\mu)$ is the angle-dependent externally incident flux. The quantity $\alpha^*(\mu)$ describes the degree of polarization at a given angle. Natural (unpolarized) radiation corresponds to $\alpha^*=0$ and $\alpha^*=\pm 1$ represent the two polarization extremes.

Using the Pomraning–Eddington approximation [8] for both the total intensity and the difference function yields

$$\begin{aligned}&I(x,\mu)=\varepsilon_1(x,\mu)E_1(x)+O_1(x,\mu)F_1(x)\\&K(x,\mu)=\varepsilon_2(x,\mu)E_2(x)+O_2(x,\mu)F_2(x)\end{aligned} \tag{5}$$