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Solving linear and nonlinear Abel fuzzy integral equations by homotopy analysis method

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Abstract

The main purpose of this article is to present an approximation method for solving Abel fuzzy integral equation in the most general form. The proposed approach is based on homotopy analysis method. This method transforms linear and nonlinear Abel fuzzy integral equations into two crisp linear and nonlinear integral equations. The convergence analysis for the proposed method is also introduced. We give some numerical applications to show efficiency and accuracy of the method. All of the numerical computations have been performed on a computer with the aid of a program written in Matlab.

Keywords: Fuzzy number; Fuzzy integral equation; Abel fuzzy integral equations; Homotopy analysis method

1. Introduction

Fuzzy integral equations are important in studying and solving a large proportion of the problems in many topics in applied mathematics, in particular in relation to physics, geographic, medical and biology. Usually in many applications, some of the parameters in our problems are represented by fuzzy number rather than crisp, and hence it is important to develop mathematical models and numerical procedures that would appropriately treat general fuzzy integral equations and solve them.

The concept of integration of fuzzy functions was first introduced by Dubois and Prade [1]. Alternative approaches were later suggested by Goetschel and Voxman [2], Kaleva [3], Nanda [4] and others. While Goetschel and Voxman [2] preferred a Riemann integral type approach, Kalva [3] defined the integral of fuzzy function, using the Lebesgue type concept for integration. One of the first applications of fuzzy integration was given by Wu and Ma [5], who investigated the fuzzy Fredholm integral equation of the second kind (FFIE-2). This work which established the existence of a unique solution for (FFIE-2) was followed by other works such as Mirzaee et al. [6] and Nguyen [7] where an original fuzzy

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differential equation is replaced by a fuzzy integral equation. Recently Liao, in his Ph.D. thesis [8], has proposed the homotopy analysis method (HAM) to solve some classes of nonlinear equations. Step by step, the method was developed and its effectiveness was proved in handling nonlinear equations [8–11].

Abel integral equations occur in many branches of scientific fields, such as microscopy, seismology, radio astronomy, electron emission, atomic scattering, radar ranging, plasma diagnostics, X-ray radiography, and optical fiber evaluation [12].

Recently, Mirzaee et al. [13–15] have studied the numerical solutions of the Fredholm fuzzy integral equations. Since the homotopy analysis method is a powerful device for solving a wide variety of problems arising in many scientific applications, we will develop the numerical methods for the approximate solutions of linear and nonlinear Abel fuzzy integral equations.

The structure of this paper is organized as follows: in Section 2, some basic definitions and results which will be used later are given. In Section 3, Abel fuzzy integral equations are introduced. In Section 4, we apply homotopy analysis method to solve Abel fuzzy integral equations, then the proposed method is implemented for solving three illustrative examples in Section 5 and finally, conclusion is drawn in Section 6.

2. Preliminaries

We now recall some definitions needed through the paper.

Definition 1. (Kaleva [3]). A fuzzy number is a fuzzy set $\upsilon : R^1 \to I = [0, 1]$ which satisfies

- υ is upper semi continuous,
- $\upsilon(x) = 0$ outside some interval $[c, d]$,
- There are real numbers $a,b : c \leq a \leq b \leq d$ for which
- $\underline{\upsilon}(x)$ is monotonic increasing on $[c, a]$,
- $\overline{\upsilon}(x)$ is monotonic decreasing on $[b, d]$,
- $\upsilon(x) = 1$, $a \leq x \leq b$.

The set of all such fuzzy number is denoted by R_F.

Definition 2. (Kaleva [3]). Let V be a fuzzy set on R. V is called a fuzzy interval if:

- V is normal: there exists $x_0 \in R$ such that $V(x_0) = 1$.
- V is convex: for all $x, t \in R$ and $0 \leq \lambda \leq 1$, it holds that $V(\lambda x + (1-\lambda)t) \geq min\{V(x), V(t)\}$,
- V is upper semi-continuous: for any $x_0 \in R$, it holds that $V(x_0) \geq \lim_{x \to 0^{\pm}} V(x)$,
- $[V]^{\alpha} = Cl\{x \in R | V(x) > 0\}$ is a compact subset of R.

The α-cut of a fuzzy interval V with $0 < \alpha \leq 1$ is the crisp set $[V]^{\alpha} = \{x \in R | V(x) > 0\}$. For a fuzzy interval V, its α-cut are closed intervals in R. They will be denoted by them by $[V]^{\alpha} = [\underline{V}(\alpha), \overline{V}(\alpha)]$. An alternative definition or parametric form of a fuzzy number which yields the same R_F is given by Kaleva [8] as follows:

Definition 3. (Ma et al. [16]). An arbitrary fuzzy number $\tilde{u}$ in the parametric form is represented by an ordered pair of functions $(\underline{u}(r), \overline{u}(r))$ which satisfy the following requirements:

- $\underline{u}(r)$ is a bounded left-continuous non-decreasing function over [0, 1],
- $\overline{u}(r)$ is a bounded right-continuous non-increasing function over [0, 1],
- $\underline{u}(r) \leq \overline{u}(r)$, for all $0 \leq r \leq 1$.

For arbitrary fuzzy numbers $\tilde{\upsilon} = (\underline{\upsilon}(r), \overline{\upsilon}(r))$, $\tilde{w} = (\underline{w}(r), \overline{w}(r))$ and real number λ, one may define the addition and the scalar multiplication of the fuzzy numbers by using the extension principle as follows: