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Nonlinear ultrasonic waves in bubbly liquids with nonhomogeneous bubble distribution: Numerical experiments

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ABSTRACT

This paper deals with the nonlinear propagation of ultrasonic waves in mixtures of air bubbles in water, but for which the bubble distribution is nonhomogeneous. The problem is modelled by means of a set of differential equations which describes the coupling of the acoustic field and bubbles vibration, and solved in the time domain via the use and adaptation of the *SNOW-BL* code. The attenuation and nonlinear effects are assumed to be due to the bubbles exclusively. The nonhomogeneity of the bubble distribution is introduced by the presence of bubble layers (or clouds) which can act as acoustic screens, and alters the behaviour of the ultrasonic waves. The effect of the spatial distribution of bubbles on the nonlinearity of the acoustic field is analyzed. Depending on the bubble density, dimension, shape, and position of the layers, its effects on the acoustic field change. Effects such as shielding and resonance of the bubbly layers are especially studied. The numerical experiments are carried out in two configurations: linear and nonlinear, i.e. for low and high excitation pressure amplitude, respectively, and the features of the phenomenon are compared. The parameters of the medium are chosen such as to reproduce air bubbly water involved in the stable cavitation process.

1. Introduction

The effect of bubbles on the propagation of ultrasonic waves in liquid is of definitive importance in many applications. In particular, in the framework of sonochemistry, the understanding of the behaviour of ultrasonic waves in bubbly water is necessary [1].

Many applications of high-power ultrasounds are based on the presence of bubbles in liquids or on cavitation. Linear and nonlinear propagation through bubbly liquids is a current and active area of research in acoustics [2,3]. This is the case in therapeutic and/or diagnostic medical applications of ultrasound [4–7], and in underwater acoustics [8,9].

Gong and Hart [10] studied the chemical effects of the collapse of one bubble by coupling chemical kinetics in the bubble with the bubble dynamics. In sonochemistry, each bubble cannot be considered as if it was isolated. The global behaviour of the whole sonoreactor has been scarcely studied. Some complex models exist in the linear range, and they are usually used for design and development in laboratories [11–13]. Yasui et al. [14] analyzed the static spatial acoustic distribution in a three-dimensional resonator through a finite element model. The wall vibration was taken into account. However, dispersion and nonlinear effects due to the bubbles were not considered, but their presence induced an effect on attenuation. Other models that take into account the presence of bubbles also exist [15–18]. Nevertheless, these models are based on linear acoustic pressure and linear bubble vibration. Among them, the models of Refs. [16–18] also consider the geometrical complexity of the system and the Bjerknes forces. However, the effects we search for in many applications of high-power ultrasounds, and especially in sonochemistry, appear only above an intensity threshold that induces a nonlinear behaviour of the pressure field. Colonius et al. [19] studied, via a numerical model, the saturation effect of acoustic pressure in a liquid with homogeneous density excited by a harmonic wall for which the oscillation frequency is much smaller than the bubble natural frequency.

The homogeneous spatial distribution of small bubbles induces an acoustic behaviour different from that of a homogeneous liquid (decrease of phase speed, dispersive properties, increase of losses, high compressibility and nonlinear effects) [2,20,21].

Several works consider nonhomogeneous spatial distributions of bubbles. In particular, the nonlinear response and resonance effects of a bubbly layer to a harmonic wave were studied by Karpov et al. [8], in the framework of parametric generation of a low-frequency signal. This study was based on a numerical model. The system bubble – acoustic field was coupled and the dynamics of the bubble vibration did not contain the adiabatic restriction. However, only one or two frequencies were considered in the incident wave and only thin bubble layer were described. Sutin et al. [9] developed an analytical model for the nonlinear scattering.

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Lo et al. [22] studied the shielding effect of a bubble layer experimentally. They described a technique of spatial control therapy by means of the generation of walls of gas bubbles. Dähnke and Keil [23] studied the nonhomogeneous distribution of bubbles by means of a linear pressure theory. Macpherson [24] characterized a bubbly liquid via the production of bubbly layers, by measuring the response to pulses and determining the attenuation. Leighton et al. [25] proposed an inversion technique to measure the size distributions of gas bubbles in the ocean. Ref. [22] justifies the need of understanding the nonlinear ultrasonic behaviour of waves in nonhomogeneous distribution of bubbles in liquids for applications in medicine. In sonochemistry, bubbles gather forming bubbly layers in the liquid. This study is especially interesting in this framework. However, we have to note that in the context of sonochemistry, the model will have to account for many other phenomena, such as Bjerknes forces attraction, the compressibility of liquid and entropic variations in the bubble equation. It must be noticed that Bjerknes forces could change the shape of the layer because of the standing waves that are formed in the layer.

In this paper we analyze the linear and nonlinear propagation of ultrasonic waves in a nonhomogeneous distributed bubbly liquid. Air bubbly layers are assumed to be placed in water. The mathematical model describing the bubble vibration and acoustic field system is presented in Section 2. A second-order equation written in a volume formulation is considered for bubbles vibration and coupled with the linear non-dissipative wave equation (bubble vibration and acoustic field calculations are coupled). The physical hypotheses and limitations of the system are given. The numerical code chosen to solve the problem is cited as well. Section 3 presents the results and the corresponding discussions. Section 4 exposes the conclusions of this work.

2. Model

The mathematical model chosen here is the one used in Refs. [20,2,26,21]. We consider plane ultrasonic waves propagating in an air–water mixture. Air bubbles are spherical, adiabatic, and all have the same size. Acoustic attenuation and nonlinear effects are only due to the bubbles oscillations. Considering an adiabatic behaviour of air in bubbles in the model (no heat transfer between air in bubbles and surrounding water) overestimates the nonlinear effects corresponding to the inertia of the bubble vibration. In the bubble equation, the compressibility of water is neglected and implies less damping in the bubble vibration and less attenuation of the associated nonlinear effects. Some other restrictive features of the model are: bubbles are monopole; their pulsation is radially symmetric and assumed to be small; their surface tension is neglected.

In the semi-infinite space domain, bubbles are not uniformly distributed but concentrated in some regions. The density of bubbles depends on the one-dimensional space coordinate, i.e. N_g is a function of x: $N_g(x)$ (the spatial distribution is not homogeneous). For numerical purpose, the semi-infinite space domain is limited to the studied domain $\Omega = [0,L]$. The system is formed by the Rayleigh–Plesset (second-order equation for bubble dynamics) and linear non-dissipative wave equations. It is written:

$$\frac{\partial^2 p}{\partial x^2} - \frac{1}{c_{0l}^2}\frac{\partial^2 p}{\partial t^2} = -\rho_{0l} N_g(x)\frac{\partial^2 v}{\partial t^2} \quad x \in \Omega,\ t \in \mathrm{T} \tag{1}$$

$$\frac{\partial^2 v}{\partial t^2} + \delta\omega_{0g}\frac{\partial v}{\partial t} + \omega_{0g}^2 v + \eta p = a(v)^2 + b\left(2v\frac{\partial^2 v}{\partial t^2} + \left(\frac{\partial v}{\partial t}\right)^2\right) \quad x \in \Omega,\ t \in \mathrm{T} \tag{2}$$

where t is the time independent variable, T is the (bounded) time domain of the study corresponding to Ω, c_{0l} is the low amplitude sound speed of water, ρ_{0l} is the density of water at the equilibrium state, $p(x,t)$ is the acoustic pressure, $v(x,t) = V(x,t) - v_{0g}$ is the bubble volume variation, where V and $v_{0g} = 4\pi R_{0g}^3/3$ are the current and equilibrium state bubble volumes, respectively, R_{0g} is the equilibrium bubble radius, $\delta = 4\nu_l/(\omega_{0g}R_{0g}^2)$ is the viscous damping coefficient, ν_l is the cinematic viscosity of water, $\omega_{0g} = \sqrt{3\gamma_g p_{0g}/\rho_{0l}R_{0g}^2}$ corresponds to the lowest resonance frequency of the bubble, $p_{0g} = \rho_{0g}c_{0g}^2/\gamma_g$ is the atmospheric pressure of air, γ_g is the specific heats ratio of air, ρ_{0g} is the density of air at the equilibrium state, c_{0g} is the low amplitude sound speed of air, $\eta = 4\pi R_{0g}/\rho_{0l}$, $a = (\gamma_g + 1)\omega_{0g}^2/2v_{0g}$, and $b = 1/(6v_{0g})$. At the outset the following initial conditions are applied (rest state):

$$p(x\neq 0, t=0) = 0 \quad x \in \Omega \tag{3}$$

$$\frac{\partial p}{\partial t}(x\neq 0, t=0) = 0 \quad x \in \Omega \tag{4}$$

$$v(x, t=0) = 0 \quad x \in \Omega \tag{5}$$

$$\frac{\partial v}{\partial t}(x, t=0) = 0 \quad x \in \Omega \tag{6}$$

v does not need boundary conditions. The acoustic field (plane waves) is generated by a continuous time-function pressure source of amplitude p_0 at $x = 0$, and its open-field (progressive) character is imposed at $x = L$ via the other boundary condition on p [27]:

$$p(x=0,t) = p_0\sin(\omega_f t) \quad t \in \mathrm{T} \tag{7}$$

$$\frac{\partial p}{\partial x}(x=L,t) = \frac{-1}{c_{0l}}\frac{\partial p}{\partial t}(x=L,t) \quad t \in \mathrm{T} \tag{8}$$

where $\omega_f = 2\pi f$ is the excitation pulsation. The description of the terms of this set of equations can be found in Ref. [21].

The Simulation of Nonlinear Waves – Bubbly Liquid (*SNOW-BL*) code was developed to solve Eqs. (1)–(8), but with a constant value N_g, i.e. in the case of homogeneous bubble distribution. The description of the numerical model can be found in Vanhille and Campos-Pozuelo [21]. It was based on the finite-difference method in space and time (implicit and second-order scheme) [28]. Here this code is adapted to the possible variations of N_g with space in order to solve the problem treated in this paper, Eqs. (1)–(8). In particular, the case for which we are interested with is now able to be simulated, i.e. bubbly layers can be introduced into the domain Ω. In a layer, the characteristics of the medium have almost not changed, i.e. this is almost the same fluid, but its acoustic ones have changed hugely. The main effect of bubbles is to increase compressibility, even for low void fraction, and therefore, to decrease sound velocity. Based on the finite-difference method, the code requires the data of the discretization in space and time. Once these data and all the physical parameters are defined and introduced into the model, the *SNOW-BL* code gives the nonlinear solution of the problem.

3. Numerical experiments, results and discussion

Section 3 is devoted to analyse the effects caused on the (linear and nonlinear) propagation in water of plane ultrasonic waves by layers of air bubble. The position and thickness of the bubbly layer, as well as the excitation pressure amplitude are changed to carry out this analysis. The shape of the layer is also changed to consider bubbly clouds. Several layers are also considered in the last paragraph of the section. We are especially interested in investigating the influence of the layers on the nonlinear characteristic of the propagation by comparing different excitation amplitude cases.

The parameters of the medium are chosen here such as to reproduce air bubbly water in the stable cavitation process: $\rho_{0l} = 1000\ \mathrm{kg/m^3}$, $c_{0l} = 1500\ \mathrm{m/s}$, and $\nu_l = 1.4\times 10^{-6}\ \mathrm{m^2/s}$ for water; $\rho_{0g} = 1.29\ \mathrm{kg/m^3}$, $c_{0g} = 340\ \mathrm{m/s}$, and $\gamma_g = 1.4$ for air. The