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The secondary Bjerknes force between two gas bubbles under dual-frequency acoustic excitation

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ABSTRACT

The secondary Bjerknes force is one of the essential mechanisms of mutual interactions between bubbles oscillating in a sound field. The dual-frequency acoustic excitation has been applied in several fields such as sonochemistry, biomedicine and material engineering. In this paper, the secondary Bjerknes force under dual-frequency excitation is investigated both analytically and numerically within a large parameter zone. The unique characteristics (i.e., the complicated patterns of the parameter zone for sign change and the combination resonances) of the secondary Bjerknes force under dual-frequency excitation are revealed. Moreover, the influence of several parameters (e.g., the pressure amplitude, the bubble distance and the phase difference between sound waves) on the secondary Bjerknes force is also investigated numerically.

1. Introduction

Driven by acoustic waves, bubbles in a liquid will oscillate, named as "acoustic cavitation" [1,2]. Because of its unique physical complexity [3], chemical applications [4] and biomedical significance [5], effects of acoustic cavitation are being employed intensively, e.g., to measure bubble size distributions [6–8], to facilitate the chemical reactions [9–15], and to perform non-invasive therapy [16–18]. When bubbles oscillate in acoustic field, the radiation pressure generated by other cavitation bubbles can cause the mutual attraction or repulsion between bubbles. This well-known phenomenon is termed as "the secondary Bjerknes force" [19,20]. The secondary Bjerknes force, leading to the translational motion of bubbles, determines the agglomeration or dispersion of bubbles [21–25], which is essential for understanding the dynamics of bubble clouds and their applications (e.g., sonochemistry). The existence of the secondary Bjerknes force could influence the effect of cavitation in sonochemical reactors through increasing the fraction of energy transfer of the combined collapsing bubbles [13]. After agglomeration due to the secondary Bjerknes force, bubbles may deform at the early stage of collapse, which decreases the efficiency of energy converting of the cavitation as well as the efficiency of the sonochemical reactions [26,27]. The mutual interaction between bubbles can also affect the pattern of the erosion of the equipment in the ultrasonic fields [14] and affect the consequent of the acoustic cleaning [15].

The direction of the secondary Bjerknes force (i.e., the bubbles whether attracting or repulsing each other) is a paramount topic of cavitation systems. According to the linear theory [19,28,29], when the driving frequency is between the linear resonance frequencies of the two bubbles, the secondary Bjerknes force between the two bubbles is repulsive. Otherwise, the secondary Bjerknes force is attractive. The linear theory has been extended by many researchers. Zabolotskaya [21] proposed that when the two bubbles approaching each other, their resonance frequencies will be effectively increased. As a result, even though the two bubbles are both driven above their resonances, the secondary Bjerknes force could change from attractive force to repulsive force. Harkin et al. [30], Doinikov and Zavtrak [23] and Ida [31,32] have also found that the distance between the bubbles could affect the direction of the secondary Bjerknes force. The nonlinearity of the bubble oscillators is one of the primary factors leading to sign reversals of the Bjerknes force. Oguz and Prosperetti [33] reported that in particular cases, the sign of the secondary Bjerknes force is opposite to the direction predicted by the linear theory, even there is slightly nonlinear oscillation when the driving pressure amplitude is below 0.5 bar. Mettin et al. [28] found similar phenomenon in the cases of the oscillating bubbles with strong collapse under the acoustic field with a high

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Nomenclature

Roman letters

c_l	speed of sound in the liquid
$\boldsymbol{e}_{12}$	unit vector pointing from bubble 1 to bubble 2
F_B	the secondary Bjerknes force
f_B	the secondary Bjerknes force coefficient
f_s	frequency of external single-frequency sound field
f_1	frequency of external sound field of frequency ω_1
f_2	frequency of external sound field of frequency ω_2
L	separation distance between the centres of two bubbles
P_0	ambient pressure
P_e	total input power (Pa)
p_{rad1}	radiation pressure generated by the oscillations of bubble 2 at the centre of bubble 1
p_{rad2}	radiation pressure generated by the oscillations of bubble 1 at the centre of bubble 2
∇p_2	pressure gradient generated by bubble 2 at the centre of bubble 1
∇P_2	complex pressure gradient generated by bubble 2 at the centre of bubble 1
R_j	instantaneous bubble radius of bubble j
$\dot{R}_j$	first derivative of the instantaneous bubble radius of bubble j
$\ddot{R}_j$	second derivative of the instantaneous bubble radius of bubble j
R_{j0}	equilibrium bubble radius of bubble j
R_{rs}	corresponding resonance bubble radius of the driving frequency of the single-frequency excitation (m)
R_{r1}	corresponding resonance bubble radius of one component driving frequency of the dual-frequency excitation (m)
R_{r2}	corresponding resonance bubble radius of the other component driving frequency of the dual-frequency excitation (m)
t	time
T	period of bubble oscillation (s)
v_j	instantaneous volume of bubble j
V_1	complex instantaneous volume of bubble 1
x_j	non-dimensional perturbation of the instantaneous bubble radius of bubble j
$\dot{x}_j$	first time derivative of x_j of bubble j
$\ddot{x}_j$	second time derivative of x_j of bubble j

Greek letters

β_j	total damping constant of bubble j
β_{acj}	acoustic damping constant of bubble j
β_{thj}	thermal damping constant of bubble j
β_{vj}	viscous damping constant of bubble j
ε_s	non-dimensional amplitude of single-frequency driving sound field
ε_1	non-dimensional amplitude of external sound field of frequency ω_1
ε_2	non-dimensional amplitude of external sound field of frequency ω_2
κ	polytropic exponent
μ_l	viscosity of the liquid
μ_{th}	effective thermal viscosity
ρ_l	density of the liquid
σ	surface tension coefficient
ω	angular frequency of the driving sound field with single-frequency
ω_1	one angular frequency of the driving sound field of dual-frequency
ω_2	another angular frequency of the driving sound field of dual-frequency
ω_{0j}	natural frequency of bubble j

Symbols

overdot	time derivative
overbar	complex conjugate
$\langle\rangle$	time average

pressure amplitude (e.g., over 1.0 bar). Doinikov [29] and Pelekasis et al. [24] investigated the effects of the harmonics of bubble oscillations on the secondary Bjerknes force. Barbat et al. [34] identified a "periodic motion pattern" when the bubbles of equal sizes are forced near their resonance frequency. There are also other parameters influence the mutual interactions between bubbles, such as the viscosity [35] and the compressibility [36] of the liquid.

In past decades, the bubble dynamics under multi-frequency (e.g., dual- or triple-frequency) acoustic excitation have attracted much attention of researchers. Compared to the single-frequency approach, the multi-frequency approach could promote the acoustical scattering cross section [37] and the mass transfer through the bubble–liquid interface [38–39], to enhance the intensity of sonoluminescence [40–43], to increase the efficiency of sonochemical reactions [27,44–47], to improve the accuracy of ultrasound imaging [48–50] and tissue ablation [51,52]. Many parameters affect the sonochemical effects of the multi-frequency approach, such as the frequencies of the ultrasound [47,53], the amplitude ratio and phase difference between component sonic waves [54–57]. It was found that the enhancement of the bubble–bubble interaction through the Bjerknes force may be one of the underlying mechanisms of the effects of multi-frequency excitation [26,27,47]. As far as we know, the secondary Bjerknes force under multi-frequency excitation has not been revealed yet.

In the present paper, the secondary Bjerknes force between two gas bubbles in liquids excited by dual-frequency acoustic waves is studied both analytically and numerically. The paper is organized as follows. In Section 2, the basic equations are introduced together with the details of the analytical solution and the numerical simulation method. In Section 3, firstly, the results obtained by the analytical and numerical simulations are compared and the valid region of the analytical solution are shown; secondly, the general features of the secondary Bjerknes force under dual-frequency excitation are investigated based on the numerical simulations; thirdly, the influence of several parameters, such as the pressure amplitude, the bubble distance and the phase difference between the two sound waves, on the secondary Bjerknes force is discussed. Section 4 concludes the main findings of the present paper. Section 5 discusses the limitations of the present work.

2. Equations and solutions

In this section, the basic equations for calculating the secondary Bjerknes force under dual-frequency excitation are introduced. Then, the analytical solution and the numerical simulation for solving these equations are given.

2.1. Basic equations

Here, the mutual interaction force (e.g., the secondary Bjerknes force) between two oscillating bubbles (numbered as "bubble 1" and "bubble 2" respectively) in liquids under the dual-frequency