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Hydrodynamic approach to multibubble sonoluminescence

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ABSTRACT

The velocity profile and radiation pressure field of a bubble cluster containing several thousand micro bubbles were obtained by solving the continuity and momentum equations for the bubbly mixture. In this study, the bubbles in the cluster are assumed to be generated and collapsed synchronously with an applied ultrasound. Numerical calculations describing the behavior of a micro bubble in a cluster included the effect of the radiation pressure field from the synchronizing motion of bubbles in the cluster. The radiation pressure generated from surrounding bubbles affects the bubble's behavior by increasing the effective mass of the bubble so that the bubble expands slowly to a smaller maximum size. The light pulse width and spectral radiance from a bubble in a cluster subjected to ultrasound were calculated by adding a radiation pressure term to the Keller–Miksis equation, and the values were compared to experimental values of the multibubble sonoluminescence condition. There was close agreement between the calculated and observed values.

1. Introduction

It is well known that in multi-bubble sonoluminescence (MBSL), several thousands of micro-bubbles are generated and collapsed synchronously with an applied ultrasound [1–3]. Recent measurements on the pulse width of a cloud of bubbles subjected to ultrasound using a time-correlated single photon counting technique indicated that the bubbles in a cloud collapse simultaneously to emit a light that is synchronous with the applied ultrasound [4].

A spherical bubble cloud subjected to harmonic far-field pressure excitation was investigated by Omta [5] and D'Agosta and Brennen [6]. They revealed that the natural frequency of the bubble cluster is always less than the natural frequency of the individual bubbles. Oguz and Prosperetti [7] investigated the interaction of two 100 μm bubbles subjected to an ultrasound with a moderate amplitude. Mettin et al. [8] considered the mutual interaction between two micro bubbles ($R_0 < 10$ μm) in a strong acoustic field ($P_a > 1$ bar, $f_d = 20$ kHz). They found that the strength and direction of the secondary Bjerknes forces due to the radiation generated by other bubbles differed from the forces predicted by linear theory. Yasui et al. [9] performed numerical simulations on a system of two bubbles and considered the interactions between n numbers of bubbles. They found that the expansion of a bubble during the rarefaction phase of ultrasound was strongly reduced by the presence of other bubbles in the cluster. They also obtained the pressure field of the center of a cloud of similarly sized, homogeneously distributed bubbles which pulsated together with an applied ultrasound. An [10] obtained the radiation sound pressure from the other bubbles acting on a particular bubble in a cluster. He investigated the collective motion of similarly sized microbubbles in a cluster and found that the radiation pressure term added in the Keller–Miksis (KM) equation considerably suppresses bubble motion. Recently, Dzaharudin et al. [11] performed numerical simulations of a cluster of encapsulated microbubbles by adding the interaction term in the KM equation. They found that the oscillation amplitude of microbubbles that are close together was reduced for a given applied ultrasound power.

In this study, MBSL is studied hydro-dynamically to obtain the velocity profile and radiation pressure field by solving the continuity and momentum equations for a spherical cluster containing numerous microbubbles. The calculated pulse width and spectral radiance for a bubble with the radiation pressure added in the KM equation are compared with the measured pulse width and spectral radiance values of the MBSL.

2. Single bubble behavior in an ultrasonic field

2.1. A set of analytical solutions for the Navier–Stokes equations

The hydrodynamics related to the single bubble sonoluminescence phenomenon involves in solving the Navier–Stokes equations for the gas inside a bubble and the liquid adjacent to the

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bubble wall. The mass, momentum and energy equations for the gas inside a spherical bubble are given as;

$$\frac{\partial \rho_g}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(\rho_g u_g r^2\right) = 0, \tag{1}$$

$$\frac{\partial}{\partial t}(\rho_g u_g) + \frac{1}{r^2}\frac{\partial}{\partial r}\left(\rho_g u_g^2 r^2\right) + \frac{\partial P_b}{\partial r} = 0, \tag{2}$$

$$\rho_g C_{v,b}\frac{DT_b}{Dt} = -\frac{P_b}{r^2}\frac{d}{dr}(r^2 u_g) - \frac{1}{r^2}\frac{d}{dr}(r^2 q_r), \tag{3}$$

where r is the distance from center, ρ_g is the gas density, u_g is the gas velocity which obeys u_g (R_b, t), the bubble wall velocity, P_b is the gas pressure, $C_{v,b}$ is a constant volume specific heat and q_r is the radial component of heat flux inside a bubble.

A set of analytical solutions for the above conservation equations [12,13] is given as;

$$\rho_g = \rho_0 + \rho_r \tag{4}$$

$$u_g = \frac{\dot{R}_b}{R_b} r, \tag{5}$$

$$P_b = P_{b0} - \frac{1}{2}\left(\rho_0 + \frac{1}{2}\rho_r\right)\frac{\ddot{R}_b}{R_b} r^2, \tag{6}$$

$$T(r) = T_b(r) + T'_b(r) \tag{7}$$

where ρ_0 is the gas density at bubble center and ρ_r is the radial dependent gas density, which are given as $\rho_0 R_b^3 = const.$ and $\rho_r = ar^2/R_b^5$,respectively. The constant a is related to the gas mass inside a bubble and was taken as $-5\rho_0/(4\pi)$. P_{b0} is the gas pressure at bubble center. The linear velocity profile describing the spatial in-homogeneities inside the bubble is a crucial ansatz for the homologous motion of a spherical object, which is encountered in another energy focusing mechanism of gravitational collapse [14]. The quadratic pressure profile given in Eq. (6), was recently verified by comparisons with direct numerical simulations [15].

The temperature profile due to the uniform pressure distribution $T_b(r)$ is well known and is valid for a non-sonoluminescing gas bubble [16]:

$$T_b(r) = \frac{B}{A}\cdot\left[-1 + \sqrt{\left(1 + \frac{A}{B}T_{b0}\right)^2 - 2\eta\frac{A}{B}(T_{bl} - T_\infty)\left(\frac{r}{R_b}\right)^2}\right], \tag{8}$$

where A and B are the are the coefficients of the temperature-dependent gas conductivity which has the form $k_g = AT + B$ [17], where $\eta = (R_b/\delta)/(k_l/B)$ and k_l is the thermal conductivity of the liquid, δ is thermal boundary layer thickness. The temperature at the bubble wall, T_{bl} in Eq. (9), can be obtained from Eq. (8) with a boundary condition of Tb T_b (R_b,t) = T_{bl}. The temperature is given as

$$T_{bl} = \frac{B}{A}\left[-(1+\eta) + \sqrt{(1+\eta)^2 + 2\frac{A}{B}\left(T_{bo} + \frac{A}{2B}T_{bo}^2 + \eta T_\infty\right)}\right]. \tag{9}$$

The temperature distribution given in Eq. (8) is valid until the characteristic time for the vibrational motion of the molecules is much less than the relaxation time for the translational motion of molecules [18].

The temperature rise and subsequent quenching due to bubble wall acceleration is given by:

$$T'_b(r) = -\frac{1}{40(\gamma-1)k'_g}\left(\rho_0 + \frac{5}{21}\rho_r\right)\left[(3\gamma-2)\frac{\dot{R}_b\ddot{R}_b}{R_b^2} + \frac{\dddot{R}_b}{R_b}\right]r^4 + C(t). \tag{10}$$

The coefficient $C(t)$ may be determined from the boundary condition at the wall, $k'_g dT'_b/dr = k_l dT_l/dr$,where T_l is the quadratic temperature distribution in the thermal boundary layer with a thickness δ'. That is,

$$C = \frac{1}{20(\gamma-1)}\left[(3\gamma-2)\dot{R}_b\ddot{R}_b R_b + \dddot{R}_b R_b^2\right] \cdot\left[\frac{\delta'}{k_l}\left(\rho_0 + \frac{5}{14}\rho_{r=R_b}\right) + \frac{R_b}{2k'_g}\left(\rho_0 + \frac{5}{21}\rho_{r=R_b}\right)\right] + T_\infty. \tag{11}$$

The temperature distribution inside a bubble may be estimated from Eqs. (7), (8), and (10) with appropriated values of δ' and k'_g, where k'_g is gas conductivity in a dense plasma state [19]. The boundary layer thickness δ' may be determined from the relation $4\pi k_l R_b^2(T_{bl} - T_\infty)/\delta' = P$, where P is the power loss due to the brake radiation (bremsstrahlung) [12]. The temperature profile given in Eq. (10) yields a thermal spike when the acceleration of the bubble wall exceeds 10^{12} m/s^2 [12] while the temperature distribution given in Eq. (8) provides a background temperature.

The mass and momentum equation for the liquid adjacent to the bubble wall provides the well-known KM equation describing the motion for the bubble wall [20], which is valid when the bubble wall velocity does not exceed the speed of sound in the liquid. That is,

$$R_b\left(1 - \frac{U_b}{C_B}\right)\frac{dU_b}{dt} + \frac{3}{2}U_b^2\left(1 - \frac{U_b}{3C_B}\right) = \frac{1}{\rho_\infty}\left(1 + \frac{U_b}{C_B} + \frac{R_b}{C_B}\frac{d}{dt}\right)\left[P_B - P_s\left(t + \frac{R_b}{C_B}\right) - P_\infty\right] \tag{12}$$

where R_b is the bubble radius, U_b is the bubble wall velocity, C_B is the speed of sound in the liquid at the bubble wall, and ρ_∞ and P_∞ are the medium density and pressure, respectively. The liquid pressure on the external side of the bubble wall P_B is related to the pressure inside the bubble wall P_b by $P_B = P_b - 2\sigma/R_b - 4\mu U_b/R_b$ where σ and μ are the surface tension and dynamic viscosity of the liquid, respectively. The pressure of the driving sound field P_s may be represented by a sinusoidal function such as $P_s = -P_A \sin(\omega t)$ where P_A is the driving sound amplitude, $\omega = 2\pi f_d$ and f_d is frequency. For incompressible limit or $U_b/C_B \to 0$, the KM equation reduces to the Rayleigh–Plesset equation.

The mass and energy equation for the liquid provides a time-dependent first order equation for the thermal boundary layer thickness δ assuming a quadratic in temperature profile, which is given by [21],

$$\left[1 + \frac{\delta}{R_b} + \frac{3}{10}\left(\frac{\delta}{R_b}\right)^2\right]\frac{d\delta}{dt} = \frac{6\alpha}{\delta} - \left[2\frac{\delta}{R_b} + \frac{1}{2}\left(\frac{\delta}{R_b}\right)^2\right]\frac{dR_b}{dt} - \delta\left[1 + \frac{1}{2}\frac{\delta}{R_b} + \frac{1}{10}\left(\frac{\delta}{R_b}\right)^2\right]\frac{1}{T_{bl} - T_\infty} \times \frac{dT_{bl}}{dt} \tag{13}$$

where α is the thermal diffusivity of liquid. The above equation determines the heat flow rate through the bubble wall. The values for instantaneous bubble radius, bubble wall velocity and acceleration and the thermal boundary layer thickness obtained from Eqs. (12) and (13) are used to calculate the density with Eq. (4), velocity with Eq. (5), pressure with Eq. (6) and temperature profile with Eq. (7) for the gas inside the bubble without any further assumptions.

2.2. Numerical integration of bubble wall motion

The KM equation, Eq. (12), is usually integrated numerically, with being normalized by the appropriate physical variables. The