



A simple model of ultrasound propagation in a cavitating liquid. Part I: Theory, nonlinear attenuation and traveling wave generation

O. Louisnard

Centre RAPSODEE, FRE CNRS 3213, Université de Toulouse, Ecole des Mines d'Albi, 81013 Albi Cedex 09, France

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ABSTRACT

The bubbles involved in sonochemistry and other applications of cavitation oscillate inertially. A correct estimation of the wave attenuation in such bubbly media requires a realistic estimation of the power dissipated by the oscillation of each bubble, by thermal diffusion in the gas and viscous friction in the liquid. Both quantities are calculated numerically for a single inertial bubble driven at 20 kHz, and are found to be several orders of magnitude larger than the linear prediction. Viscous dissipation is found to be the predominant cause of energy loss for bubbles small enough. Then, the classical nonlinear Caflish equations describing the propagation of acoustic waves in a bubbly liquid are recast and simplified conveniently. The main harmonic part of the sound field is found to fulfill a nonlinear Helmholtz equation, where the imaginary part of the squared wave number is directly correlated with the energy lost by a single bubble. For low acoustic driving, linear theory is recovered, but for larger drivings, namely above the Blake threshold, the attenuation coefficient is found to be more than 3 orders of magnitude larger than the linear prediction. A huge attenuation of the wave is thus expected in regions where inertial bubbles are present, which is confirmed by numerical simulations of the nonlinear Helmholtz equation in a 1D standing wave configuration. The expected strong attenuation is not only observed but furthermore, the examination of the phase between the pressure field and its gradient clearly demonstrates that a traveling wave appears in the medium.

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1. Introduction

The complexity and large variety of spatial and temporal scales involved in acoustic cavitation make difficult the derivation of a full theoretical model, accounting for the coupled effects between the bubble field and the sound field. Nevertheless, considerable progress has been made in the last decade. Theoretical studies in the context of single bubble sonoluminescence have allowed to restrict the ambient size of the bubbles in the micron range, owing to surface instabilities [1–7]. This has been confirmed by 20 kHz experiments, both in single bubble [8] and multi-bubble configurations (see Ref. [9] and references therein).

Besides, a large collection of experimental observations have revealed that radially oscillating bubbles in high-intensity acoustic fields tend to self-organize into bubble structures, which shapes depend on the experimental configuration, with possibly two structures or more appearing simultaneously in different zones of the liquid [10–13,9]. The shape of such structures is strongly correlated with the fundamental issue of the translational motion of the bubbles under Bjerknes forces, which have been reconsidered in the context of strongly nonlinear inertial radial oscillations and traveling waves [14,15,7].

On this basis, the action of the acoustic field on the organization of inertial bubbles has been satisfactorily described in various configurations by particle models [16,10,13,7], by calculating the forces exerted on the bubbles directly from their nonlinear dynamics. Assuming a simple shape of the sound field, some bubble structures have been remarkably caught by this method. However, the correct prediction of other structures was found to be more difficult, mainly because, as suggested by Mettin [9], the local sound field might have a complicated shape, which cannot be inferred without describing correctly the acoustic field in the medium.

The backward effect of inertial bubbles on the propagation of acoustic waves remains mainly unexplored. The main physical effects of the bubble radial oscillations on sound waves can be easily understood qualitatively. Bubbles are mechanical oscillators so that wave dispersion is expected. They oscillate non linearly for large amplitude drivings, so that waves should be nonlinear. Finally, they dissipate mechanical energy by various processes, which should produce wave attenuation. The problem has been attacked in the early work of Foldy [17] who considered linear scattering of waves by an arbitrary statistical distribution of scatterers, and obtained a linear dispersion relation. The application of this theory to the specific case of linear sound waves in bubbly liquids has been

E-mail address: louisnar@enstimac.fr

considered in Refs. [18,19]. A key feature in Foldy's approach is that for a sufficiently dilute bubbly mixture, each bubble behaves as if it were excited by the statistical average pressure field, which allows to cast aside the difficult issue of bubbles pairwise interaction. An intuitive justification of this approach can be found in Refs. [20,21].

The assumption of small amplitude waves has been relaxed by Iordansky [22] and simultaneously by van Wijngaarden [23,24] by a semi-empirical volume-averaging of the bubbly liquid equations, which is closed by a Rayleigh equation, where, as suggested by Foldy's work, the driving pressure term is the local average pressure field. The model obtained has allowed the study of nonlinear dispersive waves. The latter are classically described by the Korteweg–de Vries equation [25], and the reduction of van Wijngaarden model to the latter for moderate amplitudes has been studied by various authors both theoretically [24,26–28] and experimentally [29–31].

The popular Caflish model [32] is a rigorous generalization of Foldy's theory to the nonlinear case and yields a simplified version of van Wijngaarden model, as far as the bubbly liquid is dilute enough. The latter hypothesis has the important corollary that the mean velocity of the mixture is infinitely small, so that the momentum conservation equation coincide with the one of linear acoustics [see Eq. (2)]. A physical discussion of the latter feature can be found in Refs. [20,21].

Under the linear approximation, the Caflish model reduces to the famous dispersion relation of Foldy, which can be extended to calculate a wave attenuation coefficient, accounting for dissipation by a linearly oscillating bubble [20] and to polydisperse bubbles size distributions. Linearization allows a simple description of the sound field by an Helmholtz equation, and has been used in studies of the coupling between wave propagation and the bubble field. The gain obtained by simplifying the wave equation allows a complex description of its coupling with the bubble population evolution, spatially and along the size axis. Following such an approach, Kobelev and Ostrovski [33] have proposed an elegant model of self-action of low amplitude sound waves in bubbly liquids, accounting for the bubble drift under the action of primary Bjerknes forces and bubble coalescence favored by secondary Bjerknes forces. Although the wave equation in this study was linear, the global model was nonlinear, owing to the dependence of the wave number on the varying bubble density, which conversely evolves non linearly with the sound field. Specific solutions under different hypothesis could catch the experimentally observed self-transparency, self-focusing of sound waves in bubbly liquids, and destabilization of homogeneous bubble distributions. The latter instability has also been demonstrated in Ref. [34] by a similar approach, but involving a slightly different physics.

The attenuation of sound waves by oscillating bubbles remains normally weak for linear waves, except when the bubbles are close to the resonant size [17,35,20], which is the main cause of sound extinction considered in Ref. [33]. Since low-frequency inertial cavitation involves bubbles much smaller than the resonant size [9], the use of the linear theory of Ref. [20] predicts an abnormally low attenuation, compared to experimental data [36]. This is not astonishing since inertial bubbles typically suffer a 10-fold expansion of their radius and are expected to dissipate more energy than predicted by linear theory. Despite the latter restriction, the linear dispersion relation has often been used to predict attenuation of strong cavitation fields, because it allows the description of the problem by a linear Helmholtz equation, which is easy to solve, and allows harmonic response simulations [37–40]. Moreover, the use of a complex wave number provided by the linear dispersion relation in an Helmholtz equation somewhat masks the fact that one physical origin of wave attenuation by the bubbles is the energy dissipated by the latter. The latter point has been nicely addressed by Rozenberg [41], who restated the problem of attenu-

ation of a traveling wave by a cavitation zone in terms of energy conservation, without resorting to the linear hypothesis. The latter study made use of an empirical expression, fitted on experimental results, between the power dissipated by cavitation bubbles and the wave intensity. Doing so, realistic attenuated intensity profiles near the emitter could be calculated simply, and experimentally observed self-attenuation of the wave could be accounted for.

The last remarks suggests that the relaxation of the linear hypothesis is necessary to correctly predict attenuation by inertial bubbles, so that one should revert to the original fully nonlinear form of the Caflish model. However, although valid for any wave amplitude, the latter remains intractable for large multi-dimensional geometries, since it requires time-dependent simulations, and presents convergence problems in the range of inertial cavitation, even in 1D [42,43]. Thus, an intermediate model, simple enough to be numerically tractable, but properly accounting for the true energy dissipation by inertial bubbles, is necessary.

The motivation of this work is the derivation of such a reduced model, and can be viewed as a systematic formalization of Rozenberg's approach [41], based on the nonlinear Caflish model. The present paper extends the ideas formerly presented in Ref. [44] and is organized as follows. In Section 2, we recast the fully nonlinear Caflish equations into a mechanical energy balance equation, where we express explicitly the energy lost by the bubbly liquid on average over an oscillation period, as functions of period-averaged quantities of a single bubble dynamics. This energy loss is then computed numerically, by simulating a bubble radial dynamics equation over a typical parameter range, including the range of inertial cavitation involved in cavitation and sonochemistry experiments. In Section 3, we then seek a reduction of the Caflish equations for the main harmonic component of the acoustic field, involving the energy dissipation calculated in Section 2. Finally, in Section 4, the resulting nonlinear Helmholtz equation is solved numerically in a 1D configuration, and a detailed analysis of the obtained wave profiles is performed. The implications of the present results on the primary Bjerknes forces and 2D simulations of classical experimental configurations are deferred in a companion paper.

2. Theory

2.1. Caflish equations

The Caflish model [32] describes the propagation of an acoustic wave of arbitrary amplitude in a bubbly liquid described as a continuum, which means that the radial oscillations of all the bubbles pertaining to an elementary small volume of mixture located at a spatial point $\mathbf{r}$ can be described by a continuous spatio-temporal radius function $R(\mathbf{r}, t)$. The first two equations of the model correspond to mass and momentum conservation in the mixture:

$$\frac{1}{\rho_l c_l^2} \frac{\partial p}{\partial t} + \nabla \cdot \mathbf{v} = \frac{\partial \beta}{\partial t}, \quad (1)$$

$$\rho_l \frac{\partial \mathbf{v}}{\partial t} + \nabla p = 0. \quad (2)$$

In the above equations, $p(\mathbf{r}, t)$ is the acoustic pressure field, $\mathbf{v}(\mathbf{r}, t)$ the velocity field, ρ_l the liquid density, c_l the sound speed in the liquid, and $\beta(\mathbf{r}, t)$ is the instantaneous void-fraction, which, assuming a mono-disperse distribution of the bubbles, can be defined by:

$$\beta(\mathbf{r}, t) = N(\mathbf{r}) \frac{4}{3} \pi R(\mathbf{r}, t)^3, \quad (3)$$

where $N(\mathbf{r})$ is the local bubble density. The latter is assumed time-independent, or at least almost constant on the time scale of the oscillations. Despite the set of Eqs. (1) and (2) is very similar to

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