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Fractional-order modeling and State-of-Charge estimation for ultracapacitors

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HIGHLIGHTS

- Fractional-order modeling for ultracapacitors is systematically studied.
- The optimal model parameters are identified based on time-domain data.
- The model accuracy is confirmed though EIS tests.
- Fractional Kalman filter is formulated to track true SOC.
- The estimation performance is experimentally verified at different temperatures.

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ABSTRACT

Ultracapacitors (UCs) have been widely recognized as an enabling energy storage technology in various industrial applications. They hold several advantages including high power density and exceptionally long lifespan over the well-adopted battery technology. Accurate modeling and State-of-Charge (SOC) estimation of UCs are essential for reliability, resilience, and safety in UC-powered system operations. In this paper, a novel fractional-order model composed of a series resistor, a constant-phase-element (CPE), and a Walburg-like element, is proposed to emulate the UC dynamics. The Grünald-Letnikov derivative (GLD) is then employed to discretize the continuous-time fractional-order model. The model parameters are optimally extracted using genetic algorithm (GA), based on the time-domain data acquired through the Federal Urban Driving Schedule (FUDS) test. By means of this fractional-order model, a fractional Kalman filter is synthesized to recursively estimate the UC SOC. Validation results prove that the proposed fractional-order modeling and state estimation scheme is accurate and outperforms current practice based on integer-order techniques.

1. Introduction

Ultracapacitors (UCs), also known as supercapacitors (SCs) or Electric Double-layer Capacitors (EDLCs), have gained increasing popularity and attention as energy storage in extensive industrial fields, due to their high specific power density and extremely long cycle-life [1,2]. In order to ensure safe, reliable, and efficient operations of UC systems, precise SOC estimation plays a significant role. For instance, accurate SOC readings always facilitate an adequate exploitation of UCs power and energy without causing detrimental overcharge/overdischarge or thermal runaway [3].

Diverse SOC estimation methods have been reported including Coulomb-counting method, artificial neural networks, and model-based Kalman filter, among which model-based approaches have been the object of intense studies, owing to their excellent overall performance [4,5]. The readers can find a systematical overview of these methods in Ref. [6]. The Coulomb-counting method

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predominated in developed energy management systems, especially during initial research period, thanks to its inherent advantages of ease of implementation and high computational efficiency. However, the accuracy of this method is heavily dependent on current detection capability and initial value guess. Additionally, it tends to diverge owing to the accumulated error of current detection. Another commonly-used SOC approach is the so-called black-box models, such as artificial neural network (ANN) and fuzzy logic estimators, which can precisely depict the sophisticated nonlinear relationship between the UC SOC and its impacting factors, provided that these models are well trained offline. Nevertheless, the training process may be time-consuming and requires a large quantity of reliable data. Alternatively, online model-based SOC estimation methods have accrued tremendous attention. Particularly, the Kalman filtering-based methods have been successfully applied in a multitude of systems, due to their intrinsic traits, e.g., closed-loop self-correctness, online access, and availability of estimation error bounds [7]. The performance of this type of method, however, hinges on the model validity.

A model that can capture UC dynamics with high precision and low computation intensity is of utmost importance. There is a myriad of UC models presented in the literature, where electrochemical models (EMs) and equivalent electrical circuit models (ECMs) constitute the two dominating categories with a broad range of applications [8–11]. Based on the first principles, EMs usually utilize partial differential equations (PDEs) to explicitly depict real physical-chemical reactions inside UCs. They typically can achieve very high model accuracy at the expense of computational speed, given physical parameters credibly identified beforehand. Their complexity may hinder a practical adoption in real-time UC monitoring and management. In contrast to EMs, ECMs leverage a series of resistance-capacitance (RC) networks to simulate the electrical behaviors of UCs with much lower complexity. The performance of commonly-used ECMs, especially in terms of model accuracy and robustness, has been systematically evaluated and compared in Ref. [12]. The associated result reveals that more RC networks are usually conducive to improving model accuracy. Nevertheless, this may lead to parameter over-fitting issue. Furthermore, ECMs are often limited to describe the impedance behaviors of UCs at low frequencies, as unveiled in our previous study [13].

More recently, fractional-order models have attracted increasing interest in the arena of energy storage devices, including both batteries and UCs [14–17]. They exhibit better capability of fitting experimental data with fewer model parameters, in contrast to their integer-order counterparts. For example, Retière et al. introduced a half-order model for UCs that dramatically reduced the model order while retaining certain accuracy [18]. However, the fixation of fractional differentiation order significantly restricted the model accuracy. Martynyuk et al. presented a fractional-order model of an electrochemical capacitor, in which the model parameters were estimated by least-squares fitting of impedance data [19]. Also, N. Bertrand et al. deduced a non-linear fractional-order model from a set of linear equations resulting from a frequency analysis of UCs [20], where the model parameters were estimated in the frequency domain as well. Such a treatment proves to be reliable in a specialized laboratory environment, since the impedance spectra of UCs can be steadily and precisely obtained in a range of frequencies via Electrochemical Impedance Spectroscopy (EIS) technique. Nonetheless, the model precision may be severely compromised, when exposed to varying loading conditions in real-world operations, because the model parameters could be highly sensitive to such conditions.

In order to overcome the aforementioned downsides, this paper proposes a new fractional-order model that comprises a series resistor, a constant-phase-element (CPE), and a Walburg-like element, with the primary goal to accurately emulate the UC behavior. The Grünald-Letnikov derivative (GLD) is then adopted to perform the model discretization in a straightforward manner. The model parameters including order coefficients are calibrated using time-domain experimental data collected through the Federal Urban Driving Schedule (FUDS) test. Based on the fractional-order model, a fractional Kalman filter is further devised to track the UC SOC in real time. Extensive experimental data are applied to demonstrate the effectiveness of the developed modeling and estimation scheme. To the best of the authors' knowledge, this scheme is the first known application of fractional-order calculus to both modeling and SOC estimation in UCs.

The remainder of this paper proceeds as follows. Section 2 gives a brief induction to fractional-order calculus background and fundamentals. Section 3 elaborates the process of model characterization using time-domain data including model discretization through the GLD method. Section 4 formulates the fractional Kalman filter to estimate the UC SOC. Key conclusions are summarized in Section 5.

2. Background and fundamentals of fractional-order calculus

Fractional-order calculus (FOC) is a natural generalization of integer-order and differential calculus, which was firstly mentioned by Leibnitz and L'Hospital in their correspondence letter in 1697. By permitting integral-differential calculus operation with arbitrary or even complex order coefficients, it significantly improves modeling ability and applicability. With the continuous advance in solution methods for fractional-order differential equations, fractional-order modeling becomes a rapidly growing area of research. Meanwhile, influential studies have shown that some physical systems can be better characterized by fractional-order models, e.g., permanent magnet synchronous motors, flexible robots with viscoelastic materials, and physical systems with mass transfer and diffusion phenomena [21,22]. Hitherto, there have emerged three main definitions of FOC, namely, the Grünald-Letnikov (GL) definition, the Riemann-Liouville (RL) definition, and the Caputo definition [23]. The Grünald-Letnikov fractional-order derivative has been particularly chosen in this work, because it can effectually discretize the continuous fractional-order model in a concise and straightforward fashion.

The detailed Grünald-Letnikov derivative formulation is given as follow:

$$_{t_0}D^{\gamma}x(t) = \frac{1}{h^{\gamma}}\sum_{j=0}^{[t/h]}(-1)^j\binom{\gamma}{j}x(t-jh) \tag{1}$$

where $_{t_0}D^{\gamma}$ represents the integral-differential operator with respect to variable t, and γ is the integral-differential order ($\gamma \in R$, when $\gamma > 0$, $_{t_0}D^{\gamma}$ means fractional derivative; when $\gamma < 0$, $_{t_0}D^{\gamma}$ stands for fractional integral; when $\gamma = 0$, $_{t_0}D^{\gamma} = 1$). The initial time t_0 is always omitted in expression when it starts with 0, h denotes the sampling time interval, $[t/h]$ represents the memory length, and $\binom{\gamma}{j}$ stands for Newton binomial coefficient derived from

$$\binom{\gamma}{j} = \frac{\gamma!}{j!(\gamma-j)!} = \frac{\Gamma(\gamma+1)}{\Gamma(j+1)\Gamma(\gamma-j+1)} \tag{2}$$

with $\Gamma(\cdot)$ being the Gamma function defined by